Information Theoretic Operating Regimes of Large Wireless Networks

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***Abstract*—In analyzing the point-to-point wireless channel, insights about two qualitatively different operating regimes—bandwidth- and power-limited—have proven indispensable in the design of good communication schemes. In this paper, we propose a new scaling law formulation for wireless networks that allows us to develop a theory that is analogous to the point-to-point case. We identify fundamental operating regimes of wireless networks and derive architectural guidelines for the design of optimal schemes.**

Our analysis shows that in a given wireless network with arbitrary size, area, power, etc., there are three parameters of importance: the short-distance SNR, the long-distance SNR, and the power path loss exponent. Depending on these parameters we identify four qualitatively different regimes. One of these regimes is especially interesting since it is fundamentally a consequence of the heterogeneous nature of links in a network and does not occur in the point-to-point case; the network capacity is *both* power- and bandwidth-limited. This regime has thus far remained hidden due to the limitations of the existing formulation. Existing schemes, either multihop transmission or hierarchical cooperation, fail to achieve capacity in this regime; we propose a new hybrid scheme that achieves capacity.

I. Introduction

The classic capacity formula $C = W \log_2(1 + P_r/N_0W)$ bits/s of a point-to-point AWGN channel with bandwidth W Hz, received power P_r Watts, and white noise with power spectral density $N_0/2$ Watts/Hz plays a central role in communication system design. The formula not only quantifies exactly the performance limit of communication in terms of system parameters, but perhaps more importantly also identifies two fundamentally different operating regimes. In the power-limited (or low SNR) regime, where SNR $:= P_r/N_0W \ll 0$ dB, the capacity is approximately linear in the power and the performance depends critically on the power available but not so much on the bandwidth. In the bandwidth-limited (or high SNR) regime, where SNR $\gg 0$ dB, the capacity is approximately linear in the bandwidth and the performance depends critically on the bandwidth but not so much on the power. The regime is determined by the interplay between the amount of power and degrees of freedom available. The design of good communication schemes is primarily driven by the parameter regime one is in.

Can analogous operating regimes be identified for ad hoc wireless networks? To address this question, we are confronted with several problems. First, we have no exact formula for the capacity of networks. Second, unlike in the point-to-point case, there is no single received SNR parameter in a network.

One approach to get around the first problem is through the scaling law formulation. Pioneered by Gupta and Kumar [1], this approach seeks not the exact capacity of the network but only how it *scales* with the number of nodes in the network. The capacity scaling turns out to depend critically on how the area of the network scales with the number of nodes. Two network models have been considered in the literature. In *dense* networks [1], [2], [7], the area is fixed while the density of the nodes increases linearly with the number of nodes. In *extended* networks [3], [4], [5], [6], [7], the area grows linearly with the number of nodes while the density is fixed. For a given path loss exponent, the area of the network determines the amount of power that can be transferred across the network and so these different scalings couple the power transferred and the number of nodes in different ways.

There are two significant limitations in using the existing scaling law results to identify fundamental operating regimes of ad hoc networks. First, the degrees of freedom available in a network depends on the number of nodes in addition to the the amount of bandwidth available. By *a priori* coupling the power transferred in the network with the number of nodes in specific ways, the existing formulations may be missing out on much of the interesting parameter space. Second, neither dense nor extended networks allow us to model the common scenario where the channels between different node pairs can be in different SNR regimes. In dense networks, the channels between all node pairs are in the high SNR regime, while in extended networks, all pairs are in the low SNR regime.

In this paper, we consider a generalization that allows us to overcome these two limitations. Instead of considering a fixed area or a fixed density, we let the area of the network scale like n^ν where ν can take on any real value. Dense networks correspond to $\nu = 0$ and extended networks correspond to $\nu = 1$. By analyzing the problem for all possible values of ν, we are now considering all possible interplay between power and degrees of freedom. Note that in networks where ν is strictly between 0 and 1, channels between nodes that are far away will be at low SNR while nodes that are closer by will be at high SNR. Indeed, scaling the area A by n^ν is completely equivalent to scaling the nearest neighbor SNR as n^β, where $\beta := \alpha(1-\nu)/2$ and $\alpha \geq 2$ is the power path loss exponent.

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Note that the typical distance between nearest neighbors is of the order of $\sqrt{A/n} = n^{(\nu-1)/2}$. Since SNR is a physically more relevant parameter in designing communication systems, we will formulate the problem as scaling directly the nearest neighbor SNR.

The main result of this paper is as follows. Consider $2n$ nodes located on a regular grid of area $2A$ such that the received SNR for a transmission over the nearest neighbor distance is $\text{SNR}_s := n^\beta$. Each transmission goes through an independent uniform phase rotation. There are n sources and destination pairs, randomly chosen, each demanding the same rate. The total capacity $C_n(\alpha, \beta)$ in bits/s/Hz has a scaling exponent given by:

$$e(\alpha,\beta) := \lim_{n\to\infty} \frac{\log C_n(\alpha,\beta)}{\log n}$$
$$= \begin{cases} 1 & \beta \geq \alpha/2 - 1 \\ 2 - \alpha/2 + \beta & \beta < \alpha/2 - 1 \text{ and } 2 \leq \alpha \leq 3 \\ 1/2 + \beta & \beta \leq 0 \text{ and } \alpha > 3 \\ 1/2 + \beta/(\alpha - 2) & 0 < \beta < \alpha/2 - 1 \text{ and } \alpha > 3. \end{cases} \tag{1}$$

Note that plugging $\beta = \alpha/2$ for dense networks and $\beta = 0$ for extended networks, we recover main results of [7].

To interpret the general result (1) and to compare it to the point-to-point scenario, let us re-express the result in terms of system quantities. Recall that SNR_s is the SNR over the smallest scale in the network, which is the nearest neighbor distance. Thus, $\text{SNR}_s = n^\beta = \frac{P_r}{N_0 W}$, where P_r is the received power from a nearest neighbor node at distance $\sqrt{A/n}$ and W Hz is the channel bandwidth. Let us also define the SNR over the largest scale in the network, which is the diameter $\sqrt{A}$, as $\text{SNR}_l := n \frac{n^{-\alpha/2} P_r}{N_0 W} = n^{1-\alpha/2+\beta}$, where $n^{-\alpha/2} P_r$ is the received power from a node at distance diameter of the network. The factor n in the definition arises from the fact that there is a total of order n nodes located at a diameter distance to a given node in the network, hence n times the SNR between farthest nodes is the total SNR that can be transferred to this node over this large scale. (1) can be used to give the following approximation to the total capacity $C = W\, C_n(\alpha, \beta)$, in bits/s:

$$C \approx \begin{cases} nW & \text{SNR}_l \gg 0 \text{ dB} \\ n^{2-\alpha/2} P_r/N_0 & \text{SNR}_l \ll 0 \text{ dB and } 2 \leq \alpha \leq 3 \\ \sqrt{n} P_r/N_0 & \text{SNR}_s \ll 0 \text{ dB and } \alpha > 3 \\ \sqrt{n} W^{\frac{\alpha-3}{\alpha-2}} (P_r/N_0)^{\frac{1}{\alpha-2}} & \text{SNR}_l \ll 0 \text{ dB, } \text{SNR}_s \gg 0 \text{ dB} \\ & \text{and } \alpha > 3. \end{cases}$$

Note that there are two SNR parameters of interest in networks, the short and the long distance SNR's as opposed to the point-to-point case where there is a single SNR parameter.

The four regimes are shown in Figure 1. If the long-distance SNR is large (Reg-I), the network is in the bandwith limited regime. Long-distance communication is feasible and capacity is achieved by hierarchical cooperation and long range MIMO transmission, the scheme introduced in [7]. In all the other regimes, the long-distance SNR is less than 0 dB. The network

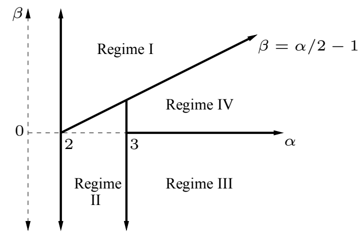

Fig. 1. The four operating regimes. The optimal schemes in these regimes are I-Hierarchical Cooperation, II-Bursty Hierarchical Cooperation, III-Multihop, IV-Multihop MIMO Hierarchical Cooperation.

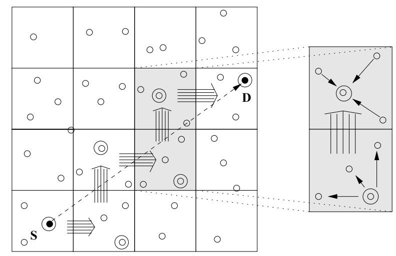

Fig. 2. The figure illustrates the optimal scheme in Regime IV. Packets are transmitted by multihopping on the network level and each hop is realized with distributed MIMO transmissions combined with hierarchical cooperation.

is power-limited and the transfer of power becomes important in determining performance. When $\alpha \leq 3$ (Reg-II), signals decay slowly with distance and the power transfer is maximized by global cooperation. This performance can be achieved by bursty hierarchical cooperation with long-range MIMO. When $\alpha > 3$, signal power decays fast with distance, and the transfer of power is maximized by cooperating in smaller scales. If the the short distance $\text{SNR}_s \ll 0$ dB (Reg-III), this scale reduces to nearest neighbors. Nearest neighbor transmissions are in the power-efficient regime and this power gain translates linearly into capacity. Classical nearest-neighbor multihop is optimal in this regime.

The most interesting case is when $\alpha > 3$ and $0 < \beta < \alpha/2 - 1$ (Reg-IV). This is the case when $\text{SNR}_s \gg 0$ dB, so nearest-neighbor transmissions are bandwidth-limited and not power-efficient in translating the power gain into capacity gain. There is the potential of improving the power transfer by cooperating in larger scales than the nearest neighbor scale. Indeed, it turns out that the optimal scheme in this regime is to cooperate hierarchically within clusters of an intermediate size, perform MIMO transmission between adjacent clusters and then multihop across several clusters to get to the final destination. (See Figure 2). The optimal cluster size is chosen such that the received SNR in the MIMO transmission is at 0 dB. The two extremes of this architecture are precisely the traditional multihop scheme, where the cluster size is 1 and the number of hops is $\sqrt{n}$, and the long-range cooperative scheme, where the cluster size is of order n and the number of hops is 1. Note also that because short-range links are bandwidth-limited and long-range links are power-limited, the network capacity is *both* bandwidth and power-limited. This regime is fundamentally a consequence of the heterogeneous nature of links in a network and does not occur in point-to-point links, nor in dense or extended networks.

II. Model

There are $2n$ nodes located on a rectangular grid with inter-node spacing d.[1] Half of the nodes are sources and the other half are destinations. The sources and destinations are randomly paired up one-to-one without any consideration on node locations. Each source has the same traffic rate R to send to its destination node and a common average transmit power budget of P Watts. The total throughput of the system is $T = nR$.

We assume that communication takes place over a flat channel of bandwidth W Hz around a carrier frequency of f_c, $f_c \gg W$. The complex baseband-equivalent channel gain between node i and node k at time m is given by:

$$H_{ik}[m] = \sqrt{G}\, r_{ik}^{-\alpha/2} \exp(j\theta_{ik}[m]) \tag{2}$$

where r_{ik} is the distance between the nodes, $\theta_{ik}[m]$ is the random phase at time m, uniformly distributed in $[0, 2\pi]$. $\{\theta_{ik}[m]\}$ are i.i.d random processes across all i and k and vary in a stationary ergodic manner over the duration of communication. We assume that they are known at all the nodes. The parameters G and $\alpha \geq 2$ are assumed to be constants; α is called the power path loss exponent. The received signal at each node is corrupted by white circularly symmetric Gaussian noise of power spectral density $N_0/2$.

III. Cutset Upper Bound

We consider a cut dividing the network area into two equal halves. We are interested in upper bounding the sum of the rates of communications $T_{L\to R}$ passing through the cut from left to right. These communications with source nodes located on the left and destination nodes located on the right half domain are depicted in bold lines in Fig. 3. Since the S-D pairs in the network are formed uniformly at random, $T_{L\to R}$ is equal to $1/4$'th of the total throughput T with high probability (w.h.p.).The maximally achievable $T_{L\to R}$ is bounded above by the capacity of the MIMO channel between all nodes S located to the left of the cut and all nodes D located to the right. Under the fast fading assumption, we have

$$T_{L\to R} \leq \max_{\substack{Q(H)\geq 0 \\ \mathbb{E}(Q_{kk}(H))\leq P/W,\, \forall k\in S}} \mathbb{E}\left(W \log\det(I + \frac{1}{N_0} H Q(H) H^*)\right) \tag{3}$$

and the entries of the $n \times n$ matrix H are given by

$$H_{ik} = \frac{\sqrt{G}\, e^{j\,\theta_{ik}}}{\left(((i_x + k_x - 1)d)^2 + ((i_y - k_y)d)^2\right)^{\alpha/4}}$$

where we use double indices to index the nodes on the rectangular grid. A left-hand side node $k \in S$ is located at position $((-k_x + 1)d, k_y d)$ and a right-hand side node $i \in D$ is located at position $(i_x d, i_y d)$ where $k_x, k_y, i_x, i_y \in \{1, \ldots, \sqrt{n}\}$. The mapping $Q(\cdot)$ is from the set of possible channel realizations H to the set of positive semi-definite transmit covariance matrices. The diagonal element $Q_{kk}(H)$ corresponds to the power allocated to the kth node at channel state H. Let us simplify notation by defining

$$\mathrm{SNR}_s := \frac{GP}{N_0 W d^\alpha} \tag{4}$$

which corresponds to the SNR between nearest neighbor nodes and

$$\hat{H}_{ik} := \frac{e^{j\,\theta_{ik}}}{\left((i_x + k_x - 1)^2 + (i_y - k_y)^2\right)^{\alpha/4}} \tag{5}$$

so that we can rewrite (3) in terms of these new variables as

$$T_{L\to R} \leq \max_{\substack{Q(\hat{H})\geq 0 \\ \mathbb{E}(Q_{kk}(\hat{H}))\leq 1,\, \forall k\in S}} \mathbb{E}\left(W \log\det(I + \mathrm{SNR}_s\, \hat{H} Q(\hat{H}) \hat{H}^*)\right). \tag{6}$$

In order to upper bound (6), we will use an approach similar to the one developed in [7, Sec.V-B] for analyzing the capacity scaling of extended networks. Note that with the definition in (5) $\hat{H}$ in (6) governs an extended network with inter-node spacing 1. However, the problem in (6) is not equivalent to the classical extended setup since here we do not necessarily assume $\mathrm{SNR}_s = 1$. Indeed in this paper, we want to avoid such arbitrary assumptions on $\mathrm{SNR}_s$and characterize the whole regime $\mathrm{SNR}_s = n^\beta$ where β is any real number.

One way to upper bound (6) is through upper bounding the capacity by the total received SNR. Such an upper bound is tight only if the SNR received by each right-hand side node is small. In the extended setup, where $\mathrm{SNR}_s = 1$, the network is highly power-limited and this turns out to be the case. In the general case however, SNR_s can be arbitrarily large which can result in high received SNR for certain right-hand side nodes that are located close to the cut. Hence before applying received SNR type of upper bounds, one needs to distinguish between those right-hand side nodes that receive high SNR and those that have poor power connections to the left-hand side. Let V_D denote the set of nodes located on a rectangular strip immediately to the right of the cut. Formally, $V_D = \{i \in D : 1 \leq i_x \leq w\}$ where $w \in \{1, \ldots, \sqrt{n}\}$ is the horizontal index of the right-most node belonging to this set. See Fig. 3. We would like to tune w so that nodes in V_D are those with high received SNR; i.e, those with received SNR larger than

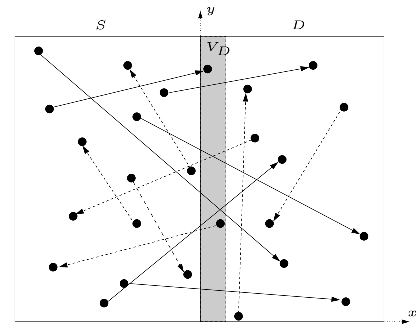

Fig. 3. The cut-set considered in Section III. The communication requests that pass across the cut from left to right are depicted in bold lines.

[1]For simplicity, we consider a regular network here, however the conclusions of the paper extend to random networks.

a threshold, say 1. Note however that we do not yet know the covariance matrix Q of the transmissions from the left-hand side nodes in (6) so we cannot compute the SNR received by a right-hand side node. For the purpose of determining V_D however, let us arbitrarily look at the case when Q is the identity matrix and define the received SNR of a right-hand side node $i \in D$ when left-hand side nodes are transmitting *independent* signals at full power to be

$$\text{SNR}_i := \frac{P}{N_0 W} \sum_{k \in S} |H_{ik}|^2 = \text{SNR}_s \sum_{k \in S} |\hat{H}_{ik}|^2 = \text{SNR}_s \, d_i \tag{7}$$

where we have defined

$$d_i := \sum_{k=1}^{n} |\hat{H}_{ik}|^2 = \sum_{k_x, k_y = 1}^{\sqrt{n}} \frac{1}{((i_x + k_x - 1)^2 + (i_y - k_y)^2)^{\alpha/2}}.$$

Simple manipulations (see [7, Lemma 5.4]) show that a good approximation for d_i is $d_i \approx i_x^{2-\alpha}$ where recall that $i_x \in \{1, \ldots, \sqrt{n}\}$ is the horizontal index of node i. Using this approximation in (7), we can identify three different regimes and specify w accordingly:

1) If $\text{SNR}_s \geq n^{\alpha/2-1}$, then $\text{SNR}_i \gtrsim 1, \forall i \in D$. Thus, let us choose $w = \sqrt{n}$ or equivalently $V_D = D$.
2) If $\text{SNR}_s < 1$, then $\text{SNR}_i \lesssim 1, \forall i \in D$. Thus, let us choose $w = 0$ or equivalently $V_D = \emptyset$.
3) If $1 \leq \text{SNR}_s < n^{\alpha/2-1}$, then let us choose
$$w = \begin{cases} \sqrt{n} & \text{if} \quad \alpha = 2 \\ \text{SNR}_s^{\frac{1}{\alpha-2}} & \text{if} \quad \alpha > 2 \end{cases}$$
so that we ensure $\text{SNR}_i \gtrsim 1, \forall i \in V_D$.

We now would like to break the information transfer from S to D in (6) into two terms: the first term governs the information transfer from S to V_D, the second term governs the information transfer from S to $D \setminus V_D$. Formally, we proceed by applying the generalized Hadamard's inequality which yields

$$\begin{aligned} T_{L \to R} \leq & \max_{\substack{Q(\hat{H}_1) \geq 0 \\ \mathbb{E}(Q_{kk}(\hat{H}_1)) \leq 1, \forall k \in S}} \mathbb{E}\left(W \log\det(I + \text{SNR}_s \hat{H}_1 Q(\hat{H}_1) \hat{H}_1^*)\right) \\ & + \max_{\substack{Q(\hat{H}_2) \geq 0 \\ \mathbb{E}(Q_{kk}(\hat{H}_2)) \leq 1, \forall k \in S}} \mathbb{E}\left(W \log\det(I + \text{SNR}_s \hat{H}_2 Q(\hat{H}_2) \hat{H}_2^*)\right) \end{aligned} \tag{8}$$

where $\hat{H}_1$ and $\hat{H}_2$ are obtained by partitioning the original matrix $\hat{H}$: $\hat{H}_1$ is the rectangular matrix with entries $\hat{H}_{ik}, k \in S, i \in V_D$ and $\hat{H}_2$ is the rectangular matrix with entries $\hat{H}_{ik}, k \in S, i \in D \setminus V_D$.

The first term in (8) can be bounded by considering the sum of the capacities of the individual MISO channels between nodes in S and each node in V_D. Using also the fact that the received SNR by each node in V_D is smaller than $n \, \text{SNR}_s$, we get

$$\begin{aligned} & \max_{\substack{Q(\hat{H}_1) \geq 0 \\ \mathbb{E}(Q_{kk}(\hat{H}_1)) \leq 1, \forall k \in S}} \mathbb{E}\left(W \log\det(I + \text{SNR}_s \hat{H}_1 Q(\hat{H}_1) \hat{H}_1^*)\right) \\ & \qquad \leq \quad w \sqrt{n} \, W \log(1 + n \, \text{SNR}_s) \end{aligned} \tag{9}$$

where we use the fact that the number of nodes in V_D is $w \sqrt{n}$.

The second term in (8) is the capacity of the MIMO channel between nodes in S and nodes in $D \setminus V_D$. Bounding the capacity with the total received SNR in the MIMO transmission, we get

$$\max_{\substack{Q(\hat{H}_2) \geq 0 \\ \mathbb{E}(Q_{kk}(\hat{H}_2)) \leq 1}} \mathbb{E}\left(W \log\det(I + \text{SNR}_s \hat{H}_2 Q(\hat{H}_2) \hat{H}_2^*)\right) \leq n^{\epsilon} \, W \, \text{SNR}_{tot}$$

for any $\epsilon > 0$ w.h.p, where

$$\text{SNR}_{tot} = \sum_{i \in D \setminus V_D} \text{SNR}_i = \text{SNR}_s \sum_{i \in D \setminus V_D} d_i. \tag{10}$$

The inequality is proved in [7, Lemma 5.2] and is precisely showing that an identity covariance matrix is good enough for maximizing the power transfer from the left-hand side. Note that SNR_i in (10) is already defined in (7) to be the received SNR of node i under *independent* signalling from the left-hand side. Using $d_i \approx i_x^{2-\alpha}$, it is easy to find an approximation for the summation $\sum_{i_y=1}^{\sqrt{n}} \sum_{i_x=w+1}^{\sqrt{n}} d_i$ in (10). Here we state a precise result that can be found by straight forward modifications of the analysis in [7]:

$$\text{SNR}_{tot} \leq \begin{cases} K_1 \, \text{SNR}_s \, n \log n & \alpha = 2 \\ K_1 \, \text{SNR}_s \, n^{2-\alpha/2} & 2 < \alpha < 3 \\ K_1 \, \text{SNR}_s \sqrt{n} \log n & \alpha = 3 \\ K_1 \, \text{SNR}_s \, (w+1)^{3-\alpha} \sqrt{n} & \alpha > 3. \end{cases} \tag{11}$$

where $K_1 > 0$ is a constant independent of SNR_s and n.

Combining the upper bounds we derived on the terms in (8) together with our choices for w specified earlier, one can get an upper bound on $T_{L \to R}$ in terms of SNR_s and n. Here, we state the final result in terms of scaling exponents: Let us define

$$e(\alpha, \beta) := \lim_{n \to \infty} \frac{\log T}{\log n} = \lim_{n \to \infty} \frac{\log T_{L \to R}}{\log n}. \tag{12}$$

and similarly $\beta := \lim_{n \to \infty} \frac{\log \text{SNR}_s}{\log n}$. We have,

$$e(\alpha, \beta) \leq \begin{cases} 1 & \beta \geq \alpha/2 - 1 \\ 2 - \alpha/2 + \beta & \beta < \alpha/2 - 1 \text{ and } 2 \leq \alpha < 3 \\ 1/2 + \beta & \beta \leq 0 \text{ and } \alpha \geq 3 \\ 1/2 + \beta/(\alpha - 2) & 0 < \beta < \alpha/2 - 1 \text{ and } \alpha \geq 3 \end{cases} \tag{13}$$

where we identify four different operating regimes depending on α and β.

IV. Order Optimal Communication Schemes

In this section, we search for communication schemes whose performance meets the upper bound derived in the previous section. We start by evaluating the performance of known schemes in the literature. The discussion reveals that existing schemes are insufficient in meeting the upper bound in the fourth operating regime in (13) which indeed lies in the most interesting parameter space. We complete this gap by introducing a new scheme in Section IV-B.[2]

[2] In different context, a similar scheme has been suggested recently in an independent work [8].

A. Known Schemes in the Literature

The derivation of the upper bound suggests the following schemes as natural candidates for optimal performance: The multihop scheme for the third regime in (13) and the hierarchical cooperation scheme for the first and second regimes in (13). The multihop scheme is based on multihopping packets via nearest neighbor transmissions. Its scaling exponent $e_{multihop}(\alpha,\beta)$ defined analogously to (12), is given by

$$e_{multihop}(\alpha,\beta) = \begin{cases} 1/2 & \beta > 0 \\ 1/2+\beta & \beta \le 0 \end{cases} \tag{14}$$

As expected, multihop only achieves the upper bound in (13) in the third regime when $\beta \le 0$ and $\alpha \ge 3$, that is when the network is extremely power limited.

The second scheme for wireless networks in [7] is based on a hierarchical cooperation architecture that performs distributed MIMO transmissions between clusters of nodes. The scaling exponent of hierarchical cooperation is given by

$$e_{HC}(\alpha,\beta) = \begin{cases} 1 & \beta \ge \alpha/2-1 \\ 2-\alpha/2+\beta & \beta < \alpha/2-1. \end{cases} \tag{15}$$

The performance in the second line is achieved by using a bursty version of the scheme. See [7, Sec. V-A]. We see that hierarchical cooperation meets the upper bound in (13) in the first regime when $\beta \ge \alpha/2-1$, i.e., when power is not a limitation. When power is limited but $2 \le \alpha \le 3$, bursty hierarchical cooperation achieves the optimal power transfer. We see that neither multihop nor hierarchical cooperation is able to meet the upper bound in the fourth regime.

B. A Hybrid Scheme: Cooperate Locally, Multihop Globally

Let us divide our rectangular network of $2n$ nodes and inter-node spacing d into square cells of side length $L = d\,\mathrm{SNR}_s^{1/(\alpha-2)}$. Note that $L \le d\sqrt{n}$, hence this is a valid choice, if $\beta \le \alpha/2-1$. Each cell contains $M = \mathrm{SNR}_s^{1/(\alpha/2-1)}$ nodes. We transmit the traffic between the source-destination pairs in the network by multihopping from one cell to the next. More precisely let the S-D line associated to an S-D pair be the line connecting its source node to its destination node. Let the packets of this S-D pair be relayed along adjacent cells on its S-D line just like in standard multihop. See Fig 2. The total traffic through each cell is that due to all S-D lines passing through the cell, which is $O(\sqrt{nM})$. Let us randomly associate each of these $O(\sqrt{nM})$ S-D lines passing through a cell with one of the M nodes in the cell, so that each node is associated with $O(\sqrt{n/M})$ S-D lines. The only rule that we need to respect while doing this association is that if an S-D line starts or ends in a certain cell, then the node associated to the S-D line in this cell should naturally be its respective source or destination node. The nodes associated to an S-D line are those that will decode, temporarily store and forward the packets of this S-D pair during the multihop operation. The following lemma states a key result regarding the rate of transmission between neighboring cells.

Lemma 1: There exists a strategy (based on hierarchical cooperation) that allows each node in the network to relay its packets to their respective destination nodes in the adjacent cells at a rate

$$R_{relay} \ge K_3\, n^{-\epsilon}$$

for any $\epsilon > 0$ and a constant $K_3 > 0$.

In steady-state operation, the outbound rate of a relay node given in the lemma should be shared between the $O(\sqrt{n/M})$ S-D lines that the relay is responsible for. Hence, the rate per S-D pair is given by $R \ge K_3\sqrt{M}\, n^{-1/2-\epsilon}$ or equivalently, the aggregate rate achieved by the scheme is

$$T_{multihop+HC} \ge K_3\, n^{1/2-\epsilon}\, \mathrm{SNR}_s^{\frac{1}{\alpha-2}}.$$

In terms of the scaling exponent, we have

$$e_{multihop+HC}(\alpha,\beta) = 1/2+\beta/(\alpha-2) \quad \text{if } 0 < \beta \le \alpha/2-1$$

which matches the upper bound (13) in the fourth regime.

Proof of Lemma 1: Let us concentrate only on two neighboring cells in the network. (Consider for example the two cells highlighted in Fig. 2): The two neighboring cells together form a regular network of $2M$ nodes with inter-node spacing d. Recall that $M = \mathrm{SNR}_s^{1/(\alpha/2-1)}$ where SNR_s is the received SNR between nearest neighbors in this network. Note that

$$\beta_c = \lim_{M\to\infty} \frac{\log \mathrm{SNR}_s}{\log M} = \alpha/2-1,$$

that is this small network of two cells is in the high SNR regime given in (15). Let the M nodes in one of the cells be sources and the M nodes in the other cell be destinations and let these source and destination nodes be paired up randomly to form M S-D pairs. Then by the first line in (15), for any $\epsilon > 0$, hierarchical cooperation can simultaneously achieve a rate $R_{relay} \ge K_3\, M^{-\epsilon} \ge K_3\, n^{-\epsilon}$ for each of these M S-D pairs, where $K_3 > 0$ is a constant independent of n. With appropriate scheduling the traffic considered here can be used to model the hop between two adjacent cells. We skip details due to space limitations.

Note that $M = \mathrm{SNR}_s^{1/(\alpha/2-1)}$ is the largest cell size one can choose while maintaining the property $\beta_c \ge \alpha/2-1$. □

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