Information theoretic upper bounds on the capacity of large extended ad-hoc wireless networks[1]

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Abstract — **We derive an information theoretic upper bound on the maximum achievable rate per communication pair in a large extended ad-hoc wireless network. We show that under a reasonably weak assumption on the attenuation due to environment, this rate tends to zero as the number of users gets large.**

I. Introduction

An important issue concerning the feasibility of ad-hoc wireless networks on a large scale is the study of their capacity. In the seminal work of P. Gupta and P. R. Kumar [1], it has been shown that under realistic assumptions regarding state of the art wireless communications, the transport capacity of such networks does not scale with $n \cdot d$ where n is the number of users and d is the diameter of the network. A first confirmation of this result from an information theoretic point of view, that is, without any assumption on the way communications take place, has been obtained in [2]. It was however assumed in there that signals are strongly attenuated over distance (power decay of order $\frac{1}{r^\alpha}$ with $\alpha > 6$).

The fact that the transport capacity does not scale with $n \cdot d$ implies in particular that if there are order n pairs in the network willing to establish communication at a common rate R and if we assume that the pairs are chosen at random, without any consideration on the users' respective locations (so the average distance between paired users is of order d), then the maximum achievable R decreases to zero as n gets large. Our aim in the present paper is to give an information theoretic proof of this fact under a weaker assumption on attenuation (power decay of order $\frac{1}{r^\alpha}$ with $\alpha > 2$).

II. Bounding technique

Let us consider a network of n users spread uniformly in a rectangular region $\Omega_n = [-\sqrt{n}, \sqrt{n}] \times [0, \sqrt{n}]$ and divided into two groups, so that each user of the first group wishes to establish communication with a correspondent chosen at random in the second group (without any consideration on their respective locations). We assume that there is no fixed infrastructure that helps relaying communications, but we also assume no restriction on the kind of help the users can give to each other; in particular, any user may act as a relay for the communicating pairs. We moreover assume that each user can transmit with power $\frac{P}{r^\alpha}$ over a distance r, where $\alpha > 2$.

Let us now divide the region Ω_n into two equal left and right parts. Statistical considerations lead to the conclusion that there are about $n/8$ communications which need to be established from left to right across the boundary. Assuming that only these $n/8$ communications need to be established and that n "mirror" imaginary users help relaying communications, we obtain, using a classical cut-set argument, that the maximum achievable rate R per communication pair in the network is bounded above by $\frac{C_n}{n/8}$, where C_n is the capacity of the following vector channel:

$$Y_j = \sum_{i=1}^n G_{ij}\, X_i + Z_j, \quad j = 1, \ldots, n,$$

where $G_{ij} = 1/|x_i - y_j|^{\alpha/2}$, $Z = (Z_j)$ is a vector of i.i.d. complex $\mathcal{N}(0,1)$ random variables, modelling surrounding noise, and x_j, y_j are the positions of the users on the left and right-hand side, respectively. We moreover consider the following relaxed power constraint:

$$\sum_{i=1}^n E(|X_i|^2) \le nP.$$

Under these assumptions, the capacity C_n is given by

$$C_n = \max_{P_k \ge 0 : \sum_k P_k \le nP} \sum_{k=1}^n \log(1 + P_k\, \lambda_k^2)$$

where λ_k are the eigenvalues of G, which can be shown to be non-negative in the present setting. Using Hadamard's inequality, we obtain the following upper bound on C_n:

$$C_n \le 2 \sum_{k=1}^n \log(1 + \sqrt{nP}\, G_{kk}),$$

and it can be shown in turn that this expression does not scale with n, so $\lim_{n\to\infty} \frac{C_n}{n} = 0$ and the maximum achievable rate R per communication pair in the network decreases to zero as n gets large.

III. Perspectives

One should notice that the result obtained here is only valid for extended networks, that is, networks with a constant density of users. A detailed study of the eigenvalues is required in order to get a conclusion in the general case. Another interesting issue is the study of the transport capacity under the assumption on attenuation made in the present paper.

References

[1] P. Gupta, P. R. Kumar, "The capacity of wireless networks", *IEEE Trans. on Information Theory*, Vol. 46(2), pp. 388–404, 2000.

[2] P. R. Kumar, L.-L. Xie, "A network information theory for wireless communications: scaling laws and optimal operation", preprint.

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