Diversity Analysis of Free-Space Optical Networks with Multihop Transmissions

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***Abstract*—Diversity analysis of multihop free-space optical (FSO) transmission shows the efficiency of this scheme to mitigate atmospheric turbulence. The significant performance of serial relaying is fundamentally a consequence of the distance-dependent channel variance. This prompts us to further investigate the requirements of different FSO networks to achieve a given diversity order. In other words, we aim at converting a network with multiple source and destination pairs to serially relayed transmissions. This indeed requires assisting nodes and *reasonably* efficient routing protocols. Besides presenting diversity analysis of multihop FSO transmission, this paper investigates how the number of relay nodes scales in terms of the number of source-destination pairs with respect to the geometry of the network and the required diversity order. Moreover, it proposes an optimal routing strategy for specific FSO networks with predefined placement of relay nodes.**

***Keywords*—Free-space optics, diversity order, multihop transmission, routing, fading channels.**

I. Introduction

The past decade saw a growing interest in improving cooperation in the context of free-space optical (FSO) communication [1], [2], [3], [4], which in turn attracted a significant attention as a potential solution for the "last mile" problem [5]. Free-space optics is a terrestrial line-of-sight (LOS) technology that enables optical transmission through the atmosphere [5] using either lasers or light-emitting diodes (LEDs) as transmit devices [6]. FSO systems offer important operational and technical assets such as immunity to electromagnetic interference, higher bandwidth capacity, and license-free long-range operation. However, the fact that the outer space forms the medium of transmission renders it unpredictable and prone to atmospheric turbulence and changing weather conditions. Fading is the major limiting factor that severely impairs FSO link performance. To combat atmospheric turbulence, researchers investigated different fading mitigation techniques. For instance, Multiple-Input and Multiple-Output (MIMO) techniques studied in the context of radio-frequency (RF) communications were extended and tailored to FSO systems [7], [8], [9]. However, FSO links are very directive and lack the wide beamwidth property of the RF transmission. This means that the presence of a small obstacle might induce large fades on all source-detector sub-channels simultaneously. Consequently, the high performance gains promised by MIMO-FSO systems is too optimistic and difficult to achieve in practice [7]. On the other hand, relay-assisted transmission emerged as a robust paradigm and a promising fading mitigation tool capable of leveraging the performance of FSO systems.

Multihop transmission and cooperative diversity are the two main cooperation schemes. In this work, we focus on multihop transmission for it exploits the distance-dependent variance of the fading amplitude and significantly boosts the performance of FSO links. As in [10], we present a diversity analysis of the multihop transmission scheme. In this paper however, the diversity gain analysis focuses on the dominant outage term corresponding to the longest FSO link to obtain tight lower and upper bounds on the overall outage probability. Indeed, this analysis shows the efficiency of serial relaying to achieve a high diversity gain. For this reason, it is perfectly reasonable to encourage multihop transmission in different FSO networks with multiple source-destination (S-D) pairs. To serve this goal, we study the relaying requirements of these networks and suggest appropriate routing protocols.

The paper is organized as follows: Section II describes the system model by identifying the modulation scheme, the communication channel, and the network topologies we consider. Section III derives the outage probability of the serial FSO network and analyzes its diversity order. Finally, in Section IV, we study how the number of relay nodes scales in terms of the number of S-D pairs to achieve a given diversity order. Moreover, we propose an optimal routing strategy for specific FSO networks with a given set of relay nodes.

II. System Model

In this section, we briefly present the transmission and the channel models. We also describe the different network topologies that we study in this paper. Note that throughout the paper, $\log$ and $\ln$ operations are with respect to base 2 and base e, respectively.

A. Transmission and Channel Models

We consider intensity-modulation and direct-detection (IM/DD) FSO systems that deploy the M-ary pulse position modulation (PPM). This means that the optical transmitter is "on" during the interval containing the pulse and "off" otherwise. If symbol k is transmitted, which means that the pulse is at the k-th slot of the M-ary PPM symbol, then the optical power incident on the photodetector during the i-th time slot, $i \in \{1, \ldots, M\}$, is given by

$$P = \begin{cases} GP_r + P_b & \text{if } i = k \\ P_b & \text{otherwise} \end{cases} \quad (1)$$

where G is the channel gain of the FSO link, P_r and P_b are the optical signal power and the background radiation power incident on the receiver, respectively. We assume that signal and background radiation dependent shot noise is dominant with respect to "dark current" and other noise components.

With M-ary PPM symbols and IM/DD FSO channels, the average number of photoelectrons per slot generated by the incident light signal (res. background radiation) λ_s (resp. λ_n) is given by:

$$\lambda_s = \eta \frac{P_r T_s / M}{hf} = \eta \frac{E_s}{hf}; \qquad \lambda_n = \eta \frac{P_b T_s / M}{hf} \tag{2}$$

where T_s is the symbol duration, η is the detector's quantum efficiency, h is Planck's constant, f is the optical center frequency, and $E_s = P_r T_s / M$ is the received optical energy per M-ary PPM symbol.

Denote by $\mathbf{V} = [V^{(1)}, ..., V^{(M)}]$ the M-dimensional decision vector, where $V^{(m)}$ corresponds to the number of photoelectrons detected in the m-th PPM slot. For a PPM symbol k (pulse at the k-th slot) sent over the described FSO channel, $V^{(m)}$ can be modeled as a Poisson random variable with parameter

$$\mathbb{E}V^{(m)} = \eta \frac{P T_s / M}{hf} = \begin{cases} G\lambda_s + \lambda_n & \text{if } m = k \\ \lambda_n & \text{otherwise.} \end{cases} \tag{3}$$

Finally, the channel gain G can be formulated as

$$G = \beta a^2, \tag{4}$$

where a is the amplitude of the turbulence-induced fading and β is the normalized path loss for an FSO link of length d with respect to a reference link of length d_0, and it is given by

$$\beta = \left(\frac{d_0}{d} \right)^2 e^{-\delta(d - d_0)} \tag{5}$$

where δ is the attenuation coefficient.

For the turbulence-induced fading coefficient a, we adopt the log-normal channel model which presents a good fit for weak turbulence regimes [4]. Therefore, the probability density function of a is given by

$$f(a) = \frac{1}{\sqrt{2\pi}\sigma a} e^{-\frac{(\ln a - \mu)^2}{2\sigma^2}} \tag{6}$$

where the parameters μ and σ satisfy the relation $\mu = -\sigma^2$ to ensure a mean path intensity of 1 [11]; i.e. $\mathbb{E}a^2 = 1$. The distance-dependent log-amplitude variance is given by [12]

$$\sigma^2 = 0.124 k^{7/6} C_n^2 d^{11/6} \tag{7}$$

where k and C_n^2 are the wave number and the refractive index structure constant, respectively.

B. Network Topology

We consider FSO networks with different node topologies. In all these networks however, there are N randomly chosen S-D pairs willing to establish communication with the help of a given number of relay nodes. Each FSO node is equipped with an FSO transceiver that can transmit directionally within an infinitesimally small angle and can receive omnidirectionally. The orientation of any FSO transmitter can be steered to any possible direction.

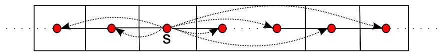

Fig. 1. All possible transmissions starting at the specified source node S.

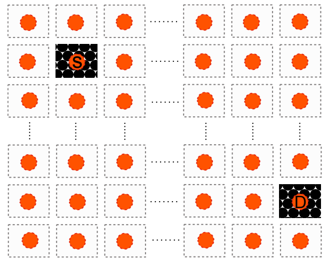

Fig. 2. Randomly chosen destination D for the specified source node S.

The problem of handling communication between N uniformly chosen S-D pairs has already been largely addressed in the context of RF wireless networks (see [13] for a seminal paper on the subject). While interference is a major issue for RF networks, it is absent in the context of FSO networks, because FSO systems are perfectly directional. For the very same reason however, FSO nodes cannot broadcast information to many nodes at once, nor can the communication be redirected during the time of transmission, for efficiency reasons.

We consider two different scenarios, namely, FSO networks with varying number of relay nodes and FSO networks with a given set of relay nodes.

1) Networks with a Varying Number of Relay Nodes: These networks are characterized by the scaling of the number of helping nodes in the network in terms of the number of transmissions. They are made up of N square cells of area 1 willing to communicate with each other in a random fashion.

a) One-Dimensional Network: A one-dimensional network consists of N square cells of area 1 placed in a linear fashion. Ignoring the relay nodes, each cell has one FSO node willing to communicate data to a random destination cell. Therefore, without the assisting nodes, there are exactly N FSO nodes placed at equal distances along a straight line of length N, as shown in Fig. 1. The number of relay nodes added to each of these N cells varies with respect to the desired performance.

b) Two-Dimensional Network: A two-dimensional network consists of N square cells of area 1 that form a square (assume $\sqrt{N}$ is integer). Ignoring the relay nodes, each cell has one FSO node willing to communicate with a randomly chosen destination cell. In other words, there are N FSO nodes uniformly distributed over a square of area N. As in the previous case, each node requires to communicate with one of the remaining nodes, as shown in Fig. 2. The number of relay nodes added to each of these N cells varies with respect to the performance requirements.

2) Two-Dimensional Network with a Given Set of Relay Nodes: Consider a third network model, which is a two-dimensional network with N source nodes placed on one side of the network and N destination nodes placed on the other side. These two groups are at a distance of N cells from each other. Each of the cells between these two groups has only one

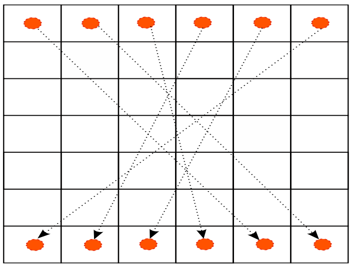

Fig. 3. Source nodes at the top have distinct destinations at the bottom. Each of the empty cells has one relay node. Note that in this example $N = 6$.

relay node. Equivalently, there are exactly $(N-1) \times N$ relay nodes in the network. See Fig. 3.

III. Diversity Analysis of Multihop Transmission

In this section, we investigate the diversity order of multihop FSO transmission assuming no background noise; i.e. $\lambda_n = 0$. Note that at high signal energies, fading and quantum-limited shot noise become the main limiting factors [7]. Each relay decodes and forwards the received M-ary PPM signal until the message reaches the destination node. As discussed in [10], the conventional definition of diversity order for log-normal channels is insignificant and it always yields infinity. For this reason, we utilize the notion of relative diversity order (RDO) that was introduced to analyze the performance of indoor radio propagations over log-normal channels [4]. For a predefined rate of transmission R, we define the relative diversity order as

$$RDO(\lambda_s) = \frac{\ln \mathbb{P}_{out}(R)}{\ln \mathbb{P}_{out(S,D)}(R)} \tag{8}$$

where $\mathbb{P}_{out}(R)$ is the outage probability of the multihop scheme and $\mathbb{P}_{out(S,D)}(R)$ is the outage probability of the direct transmission from the source node S to the destination node D with a transmit power equal to the total transmit power of the multihop scheme. We further define the asymptotic RDO as

$$ARDO = \lim_{\lambda_s \to \infty} RDO(\lambda_s).$$

Theorem 1. *The asymptotic relative diversity order of the free-space optical multihop scheme is given by*

$$ARDO = \left(\frac{d_{S,D}}{d_{\max}}\right)^{11/6}$$

where $d_{S,D}$ and $d_{\max}$ represent the distance from the source node to the destination node and the maximum distance separating any two consecutive nodes along the serial relaying scheme, respectively.

A. Outage Probability Expression

In the absence of background noise, the point-to-point FSO channel of the i-th hop reduces to an M-ary erasure channel with parameter

$$P_{e,i} = \mathbb{P}\left(V_i^{(k)} = 0 | \text{symbol } k\right) = e^{-\beta_i \alpha_i^2 \lambda_s} \tag{9}$$

where β_i and α_i are the normalized path loss and the fading amplitude of the i-th hop, respectively. Likewise, the end-to-end multihop scheme with decode-and-forward relay nodes reduces to an M-ary erasure channel with parameter

$$\gamma = 1 - \prod_{i=1}^{N} (1 - P_{e,i}) = 1 - \prod_{i=1}^{N} \left(1 - e^{-\beta_i \alpha_i^2 \lambda_s}\right) \tag{10}$$

where N is the total number of hops. As such, for a predefined rate of transmission R, the outage probability is given by

$$\mathbb{P}_{out}(R) = \mathbb{P}\left((1-\gamma)\log M \le R\right). \tag{11}$$

B. Diversity Analysis

Having defined the outage probability of the FSO multihop transmission, we can give the proof for Theorem 1. The proof relies on having tight lower and upper bounds on the outage probability in (11).

Proof: Let K represent the index of the longest hop along the multihop scheme. The lower bound on the outage probability of the serial relaying scheme with N hops is given by

$$\begin{aligned}
\mathbb{P}_{out}(R) &\ge \mathbb{P}\left(\left(1 - e^{-\min\limits_{i=1,\ldots,N}\{\beta_i\alpha_i^2\}\lambda_s}\right) \le \frac{R}{\log M}\right) \\
&= 1 - \prod_{i=1}^{N} \mathbb{P}\left(\beta_i \alpha_i^2 > \frac{1}{\lambda_s} \ln\left(\frac{\log M}{\log M - R}\right)\right) \\
&\ge \mathbb{P}\left(\alpha_K^2 \le \frac{1}{\beta_K \lambda_s} \ln\left(\frac{\log M}{\log M - R}\right)\right) \\
&= Q\left(-\frac{\frac{1}{2}\log\left(\frac{1}{\beta_K\lambda_s}\ln\left(\frac{\log M}{\log M - R}\right)\right) + \sigma_K^2}{\sigma_K}\right)
\end{aligned} \tag{12}$$

and the upper bound is given by

$$\begin{aligned}
\mathbb{P}_{out}(R) &\overset{(a)}{\le} \mathbb{P}\left(\left(1 - N \times e^{-\min\limits_{i=1,\ldots,N}\{\beta_i\alpha_i^2\}\lambda_s}\right) \le \frac{R}{\log M}\right) \\
&= 1 - \mathbb{P}\left(\min_{i=1,\ldots,N}\{\beta_i\alpha_i^2\} > \frac{1}{\lambda_s}\ln\left(\frac{N \times \log M}{\log M - R}\right)\right) \\
&= 1 - \prod_{i=1}^{N}\left(1 - \mathbb{P}\left(\beta_i\alpha_i^2 \le \frac{1}{\lambda_s}\ln\left(\frac{N \times \log M}{\log M - R}\right)\right)\right) \\
&\le 1 - \left(1 - \mathbb{P}\left(\beta_K\alpha_K^2 \le \frac{1}{\lambda_s}\ln\left(\frac{N \times \log M}{\log M - R}\right)\right)\right)^N \\
&\overset{(b)}{\approx} N \times Q\left(-\frac{\frac{1}{2}\log\left(\frac{1}{\beta_K\lambda_s}\log\left(\frac{N \times \log M}{\log M - R}\right)\right) + \sigma_K^2}{\sigma_K}\right)
\end{aligned} \tag{13}$$

where we have used the inequality $\gamma \le \sum_{i=1}^{N} e^{-\beta_i\alpha_i^2\lambda_s}$ to obtain (a) and $1 - (1 - Q(x))^N \approxeq N \times Q(x)$, for large values of x, to obtain (b). Note that $\sigma_K^2 = 0.124 k^{7/6} C_n^2 d_{\max}^{11/6}$ is the log-amplitude variance of the longest hop (K-th hop) and $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2} dt$.

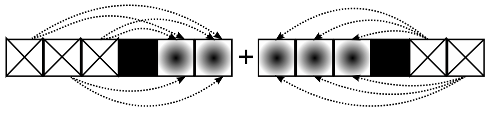

Fig. 4. Cells marked with X (resp. shady circle) represent source (resp. destination) cells. We show all the possible transmissions that require the assistance of the black cell.

Finally, the outage probability of the direct transmission is given by

$$\mathbb{P}_{out(S,D)}(R) = \mathbb{P}\left(\alpha_{S,D}^2 \leq \frac{1}{N\lambda_s}\ln\frac{\log M}{\log M - R}\right) = Q\left(-\frac{\frac{1}{2}\log\left(\frac{1}{N\lambda_s}\ln\frac{\log M}{\log M-R}\right)+\sigma_{S,D}^2}{\sigma_{S,D}}\right) \quad (14)$$

where $\alpha_{S,D}$ and $\sigma_{S,D}^2 = 0.124k^{7/6}C_n^2 d_{S,D}^{11/6}$ are the fading amplitude and the log-amplitude variance of the direct channel, respectively.

Using equations (12), (13), and (14), and $Q(x) \approxeq e^{-x^2/2}$, for large values of x, we get

$$\left(\frac{\sigma_{S,D}}{\sigma_K}\right)^2 \leq ARDO \leq \left(\frac{\sigma_{S,D}}{\sigma_K}\right)^2 = \left(\frac{d_{S,D}}{d_{\max}}\right)^{11/6}. \quad (15)$$

■

IV. Serial Relaying in Different Network Models: Relay Nodes and Routing

Based on our diversity analysis, the outage of the longest FSO link dominates the end-to-end outage probability of the serial relaying scheme and dictates the asymptotic RDO accordingly. For this reason, the ultimate goal in any given FSO network would be to have multihop transmissions from the source node to the destination node with the shortest hops possible. This also suggests placing relay nodes along the straight source-to-destination path at equal distances as proved in [10]. In this context, we can formulate two valid problems: (1) Characterize the tradeoff between the achieved asymptotic RDO and the number of relay nodes required in an FSO network. (2) Given a set of relay nodes, maximize the asymptotic RDO of all transmissions in an FSO network by optimizing the cooperation strategy (routing protocol). Both problems aim at converting FSO networks with multiple S-D pairs to serially relayed transmissions.

A. Scaling the Number of Relay Nodes

An improved asymptotic RDO naturally requires more relay nodes to establish serial transmissions with shorter hops. We are interested in the scaling of relay nodes with respect to the asymptotic RDO. We consider both one and two dimensional networks with N S-D pairs.

1) One-Dimensional Network: As described in Section II-B, a one-dimensional FSO network consists of N cells placed in a linear fashion. Each cell contains a source node willing to communicate with a destination in a randomly chosen cell.

Theorem 2. *For all the N transmissions in a one-dimensional FSO network to achieve an asymptotic RDO of $N^{11/6}$, the total number of FSO nodes should scale as N^2.*

Proof: The distance separating an S-D pair in a one-dimensional network is $\Theta(N)$ on average. Therefore, to achieve an asymptotic RDO of $N^{11/6}$, each source node must start a multihop transmission with hops of size $\Theta(1)$. In other words, all FSO links along the serial cooperation scheme should have a length of order 1. This means that each transmission requires $\Theta(N)$ relay nodes on average, which indeed implies the need for $\Theta(N^2)$ assisting nodes to serve all S-D pairs. ■

A trivial solution would be to place $\Theta(N)$ FSO nodes in each cell, which results in a total of $\Theta(N^2)$ FSO nodes. This solution is optimal in terms of the scaling of assisting nodes. However, nodes in the middle of a one-dimensional network normally experience higher traffic than those at the boundaries. Fig. 4 shows all the possible transmissions that pass through the cell marked in black. Let S_x and D_x denote the source and the destination cell indices corresponding to the x-th transmission, respectively. Since the destination nodes are chosen randomly, the mean number of transmissions N_i that cell i has to serve is given by

$$N_i = \sum_{x=1}^{N-1}\mathbb{P}(S_x < i, D_x > i) + \mathbb{P}(S_x > i, D_x < i) = \frac{2(i-1)(N-i)}{N-1} \approx \frac{2i(N-i)}{N} = O(N). \quad (16)$$

As in the trivial solution, the total number of FSO nodes in the network is given by

$$T_N = \sum_{i=1}^{N} N_i = \sum_{i=1}^{N}\frac{2(i-1)(N-i)}{N-1} = \Theta(N^2). \quad (17)$$

Note that we ignore the case where the black cell is either a source node or a destination node, because this only adds a constant term to the total number of nodes required in this cell.

To study the tradeoff between the asymptotic RDO achieved and the required number of FSO nodes, we generalize Theorem 2 in Theorem 3.

Theorem 3. *For all the N transmissions in a one-dimensional FSO network to achieve an asymptotic RDO of $(N^{1-\alpha})^{11/6}$, $0 \leq \alpha \leq 1$, the total number of FSO nodes should scale as $N^{2-\alpha}$.*

Proof: To achieve an asymptotic RDO of $(N^{1-\alpha})^{11/6}$, $0 \leq \alpha \leq 1$, each hop must have a maximum size of N^α (Recall that the distance separating an S-D pair is $\Theta(N)$ on average). Therefore, the number of hops required for each S-D pair is divided by N^α compared to the serial relaying with hops of size $\Theta(1)$. As such, the total number of FSO nodes required to achieve an asymptotic RDO of $(N^{1-\alpha})^{11/6}$ is given by

$$T_N(\alpha) = \frac{T_N}{N^\alpha} = \Theta(N^{2-\alpha}). \quad (18)$$

■

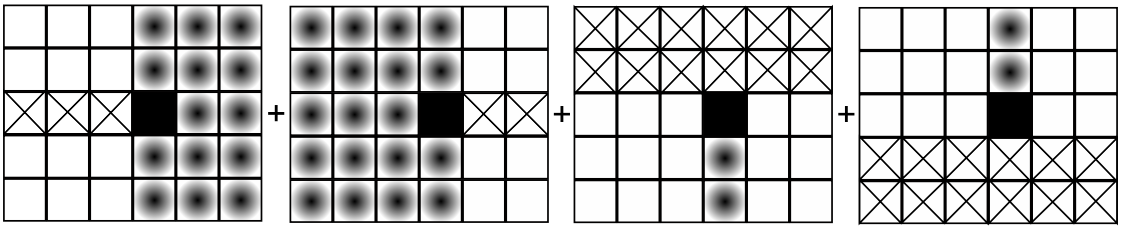

Fig. 6. Cells marked with X (resp. shady circle) represent source (resp. destination) cells. We show all S-D pairings that require the assistance of the black cell.

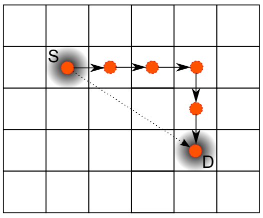

Fig. 5. Horizontal and vertical serial relaying in a two-dimensional FSO network.

For $\alpha = 1$, the asymptotic RDO is equal to 1 (only direct transmissions) and no relaying is required. In other words, we have N FSO nodes directly transmitting to randomly chosen destination cells.

2) Two-Dimensional Network: As described in Section II-B, a two-dimensional FSO network is made up of N square cells of area 1 forming a big square of area N. Each of these cells has a source node with a destination in a randomly chosen cell.

Theorem 4. *For all the N transmissions in a two-dimensional FSO network to achieve an asymptotic RDO of $(\sqrt{N})^{11/6}$, the total number of FSO nodes should scale as $N^{1.5}$.*

Proof: The distance separating any S-D pair in a two dimensional network is $\Theta(\sqrt{N})$ on average. Therefore, to achieve an asymptotic RDO of $(\sqrt{N})^{11/6}$, each source node must start a multihop transmission with hops of size $\Theta(1)$. In other words, each S-D pair requires $\Theta(\sqrt{N})$ hops, equivalently, $\Theta(\sqrt{N})$ assisting nodes. As such, the total number of relay nodes scales as $\Theta(N\sqrt{N})$. ∎

A simple routing strategy would be to multihop across all the cells along the straight line joining a source to its destination. However, this requires steering the FSO transceivers over all the angles, which in turn requires many layers of communication. On the other hand, a simpler yet order optimal routing protocol would be to have only two layers of communication, namely a horizontal layer and a vertical layer. This protocol consists of multihopping along the horizontal cells, followed by multihopping along the vertical cells, as shown in Fig. 5. For any S-D pair, this protocol results in $\Theta(\sqrt{N})$ hops of size $\Theta(1)$ along both directions. Indeed, this protocol achieves an asymptotic RDO of $(\sqrt{N})^{11/6}$.

Notice that the cells in the center of the square area experience higher traffic than those in the boundaries. Fig. 6 shows all the possible S-D pairs that require the help of the cell marked in black. Using the same approach as in the case of one dimentional network, we can show that the mean number of transmissions $N_{i,j}$ that the cell with column index i and row index j has to serve is given by:

$$N_{i,j} = \frac{2\sqrt{N}\left[i\left(\sqrt{N}-i\right)+j\left(\sqrt{N}-j\right)\right]}{N} = O(\sqrt{N}).$$

Therefore, the total number of FSO nodes in the network is given by

$$T_N = \sum_{i=1}^{\sqrt{N}}\sum_{j=1}^{\sqrt{N}} N_{i,j} = \Theta(N^{1.5}). \tag{19}$$

To study the tradeoff between the asymptotic RDO achieved and the required number of FSO nodes, we generalize Theorem 4 in Theorem 5.

Theorem 5. *For all the N transmissions in a two-dimensional FSO network to achieve an asymptotic RDO of $(N^{0.5-\alpha})^{11/6}$, $0 \leq \alpha \leq 0.5$, the total number of FSO nodes scales as $N^{1.5-\alpha}$.*

Proof: To achieve an asymptotic RDO of $(N^{0.5-\alpha})^{11/6}$, $0 \leq \alpha \leq 0.5$, each hop must have a size of N^{α}, in both horizontal and vertical directions. Therefore, the number of hops required for each S-D pair is divided by N^{α} compared to the serial relaying with hops of size $\Theta(1)$. As such, the total number of FSO nodes reduces to

$$T_N(\alpha) = \frac{T_N}{N^{\alpha}} = \Theta(N^{1.5-\alpha}). \tag{20}$$

∎

In the extreme case, where $\alpha = 0.5$, we only have direct transmissions without relaying. In this case, we need $\Theta(1)$ FSO nodes in each of the N cells.

B. Parallel Neighbor-Sort Routing for Two-Dimensional Networks with a Given Set of Relay Nodes

Given a two-dimensional network, as described in Section II-B, we propose a routing protocol that utilizes the relay nodes available in the network in an optimal way. This network has N source nodes placed on one side of the network willing to transmit data to N distinct destination nodes placed at a distance of N cells on the other side of the network. See Fig. 7. The proposed cooperation strategy relies on a sorting algorithm and aims at establishing serially relayed S-D pairs with N hops of size $\Theta(1)$.

We assign to each source node an integer value that represents the column index of the corresponding destination node. Therefore, the integers assigned to source nodes correspond

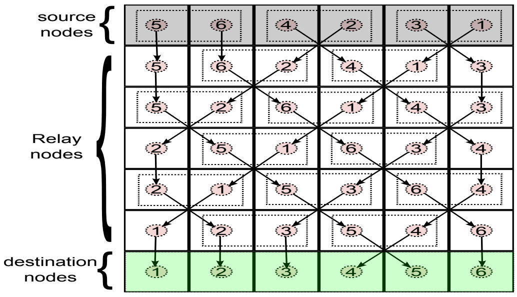

Fig. 7. Parallel neighbor-sort routing protocol.

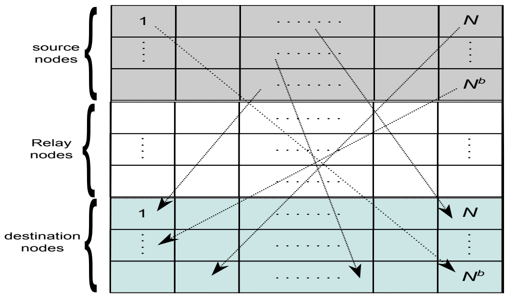

Fig. 8. A two-dimensional network with N^b S-D pairs.

to some permutation of integers from 1 to N. As such, an equivalent problem to our original routing problem would be to sort these permuted integers in N steps, such that at each step any integer can switch positions only with the neighboring integers. In the example shown in Fig. 7, source nodes 1 to 6 are willing to communicate data to destination nodes 5, 6, 4, 2, 3, and 1, respectively. The sorting algorithm goes as follows:

(1) Consider windows of size 1×2 containing two integers each.

(2) Sort the integers that belong to the same window.

(3) Shift the windows by their half size (i.e. 1) to the right in a cyclic fashion and go back to step (2).

In fact, this is known as parallel neighbor-sorting algorithm [14]. This algorithm successfully sorts any permutation of N integers in at most N steps. Equivalently, the neighbor-sort routing protocol requires at most $N \times (N-1)$ assisting nodes, which results in a total of $N \times (N+1)$ FSO nodes in the network. We run the proposed routing scheme in the example shown in Fig. 7, where we have six source nodes on one side of the square trying to communicate with six distinct destination nodes placed on the other side. In this example, the routing scheme starts with integers 4 and 2 (resp. 3 and 1) in the second (resp. third) window switching places, while integers 5 and 6 in the first window remaining in their places. The proposed strategy maximizes the asymptotic RDO of each of the N transmissions. Since we have N hops of size $\Theta(1)$ each, then the asymptotic RDO of each transmission is $N^{11/6}$. Note that the distance separating each S-D pair is $\Theta(N)$.

Generalization: In the same two-dimensional FSO network of area $\Theta(N^2)$, let us consider N^b S-D pairs placed as shown in Fig. 8, with $1 \leq b \leq 2$. In this case, the minimum hop size can be shown to be $\Theta(N^{b-1})$, leading to a maximum asymptotic RDO of order $(N^{2-b})^{11/6}$. To achieve this ARDO, we implement the parallel neighbor sort algorithm using windows of size $N^{b-1} \times 2N^{b-1}$.

V. Conclusion

Multihop transmission exploits the distance-dependent channel variance to improve the outage probability of FSO systems with respect to direct transmission. Diversity analysis shows that the asymptotic relative diversity order is dictated by the size of the longest hop. This is the motivation behind our effort to operate FSO networks with short distance multihop transmissions in order to accommodate multiple source-destination pairs. Along this line, we considered two scenarios. In the first scenario, we calculated the scaling of the number of assisting nodes needed to achieve a given asymptotic RDO. In the second scenario, we considered a two-dimensional FSO network with N S-D pairs and a predefined placement of relay nodes. The challenge resided in finding the optimal routing protocol that achieves the highest possible asymptotic RDO. For this purpose, we used the parallel neighbor-sort algorithm. Finally, we would like to point out that for an FSO network with a general topology and a predetermined set of relay nodes, how to optimize cooperation in order to accommodate multiple S-D pairs remains an open problem.

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