

## Joint distribution of subdeterminants of Wishart matrices

Let us consider  $H$  a  $3 \times 3$  matrix with i.i.d.  $\sim \mathcal{N}_{\mathbb{C}}(0, 1)$  entries. What is the joint distribution of the following 3 random variables'?

$$\left\{ \begin{array}{l} X_1 = |h_{11}|^2 \\ X_2 = |h_{11}h_{22} - h_{12}h_{21}|^2 \\ X_3 = |\det(H)|^2 = \det(HH^*) \end{array} \right.$$

Notice that  $X_i$  correspond to the modulus square of the determinant of the upper left  $i \times i$  submatrix of  $H$ .

Same question when 3 is replaced by  $n$ .

*Remark:* The answer is known for the following random variables:

$$Y_i = \det(H_i H_i^*)$$

where  $H_i$  is the  $i \times n$  upper submatrix of  $H$ . The joint distribution of the  $Y$ 's can be computed by using the Choleski decomposition of the matrix  $HH^*$ . But such a technique does not work for the  $X$ 's.