Open problem 0.1. *Let H be an $n \times (n+m)$ matrix with i.i.d.$\sim \mathcal{N}_{\mathbb{C}}(0,1)$ entries and G be its left $n \times n$ sub-matrix. Let also $\lambda_1 \geq \ldots \geq \lambda_n \geq 0$ be the eigenvalues of HH^* and $\mu_1 \geq \ldots \geq \mu_n \geq 0$ be the eigenvalues of GG^*. What is the joint distribution of $\lambda_1, \ldots, \lambda_n, \mu_1, \ldots, \mu_n$ for general values of the parameters n and m?*

For $n = 1$, the answer is easy.

For $n = 2$, we make the following conjectures.

Conjecture 0.1. *Let $n = 2$, $m = 1$, $\lambda_1 \geq \lambda_2 \geq 0$ be the eigenvalues of HH^* and $\mu_1 \geq \mu_2 \geq 0$ be the eigenvalues of GG^*. Then the joint distribution of $\lambda_1, \lambda_2, \mu_1, \mu_2$ is given by*

$$p(\lambda_1, \lambda_2, \mu_1, \mu_2) = (\lambda_1 - \lambda_2)\,(\mu_1 - \mu_2)\,\exp(-\lambda_1 - \lambda_2)\,1_{\{\lambda_1 \geq \mu_1 \geq \lambda_2 \geq \mu_2 \geq 0\}},$$

Conjecture 0.2. *Let $n = 2$, $m = 2$, $\lambda_1 \geq \lambda_2 \geq 0$ be the eigenvalues of HH^* and $\mu_1 \geq \mu_2 \geq 0$ be the eigenvalues of GG^*. Then the joint distribution of $\lambda_1, \lambda_2, \mu_1, \mu_2$ is given by*

$$\begin{aligned} &p(\lambda_1, \lambda_2, \mu_1, \mu_2) \\ &= (\lambda_1 - \lambda_2)\,(\lambda_1 - \max(\lambda_2, \mu_1))\,(\min(\lambda_2, \mu_1) - \mu_2)\,(\mu_1 - \mu_2)\,\exp(-\lambda_1 - \lambda_2)\,1_{\{\lambda_1 \geq \{{\mu_1 \atop \lambda_2}\} \geq \mu_2 \geq 0\}}, \end{aligned}$$