A generalization of Cauchy's determinant identity

Let H_k be the $n \times n$ matrix defined as

$$h_{ij} = \frac{1}{(x_i + y_j)^k}$$

where x_i, y_j are arbirary positive numbers and k is a positive integer. Cauchy's determinant identity reads

$$\det(H_1) = \frac{\Delta(\mathbf{x})\,\Delta(\mathbf{y})}{\prod_{i,j=1}^{n}(x_i + y_j)}$$

where $\Delta(\mathbf{x}) = \prod_{i<j}(x_j - x_i)$.

Borchardt then showed that

$$\det(H_2) = \det(H_1)\,\mathrm{perm}(H_1)$$

where $\mathrm{perm}(H_1)$ is the permanent of the matrix H_1. This allows to conclude that $\det(H_2)$ is of the form

$$\det(H_2) = \frac{\Delta(\mathbf{x})\,\Delta(\mathbf{y})\,P_2(\mathbf{x},\mathbf{y})}{\prod_{i,j=1}^{n}(x_i + y_j)^2}$$

where $P_2(\mathbf{x},\mathbf{y})$ is some polynomial with non-negative coefficients.

The conjecture is: for any integer $k \geq 1$, we have

$$\det(H_k) = \frac{\Delta(\mathbf{x})\,\Delta(\mathbf{y})\,P_k(\mathbf{x},\mathbf{y})}{\prod_{i,j=1}^{n}(x_i + y_j)^k}$$

where $P_k(\mathbf{x},\mathbf{y})$ is again some polynomial with non-negative coefficients.

This conjecture was made by Emmanuel Preissmann.