Spatial Coupling and the Threshold Saturation Phenomenon

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The latest version of these slides (Keynote and PDF) can be found at
https://ipg.epfl.ch/doku.php?id=en:publications:scc_tutorial

Part IV: Spatial coupling - A General Phenomenon

Outline

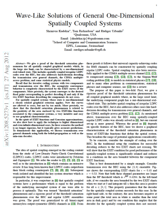
Spatial Coupling - A General Phenomenon

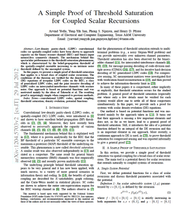
- General one-dimensional systems
- General BMS channels - Threshold Saturation and Universality
- Multi-user communications and ISI channels
  - Multi-access channels
  - Noisy Slepian-Wolf
  - Finite state channels
  - Many more...
- Problems beyond Communications
  - Compressive sensing
  - K-SAT

Practical Aspects and Open Questions

- Universality
- Windowed decoding
- Rate loss
- Scaling
- Decoding Speed
- Complexity and choice of parameters

In this part we will look at some of the applications where spatial coupling has been successfully applied. The purpose of this part is not to discuss each application in detail. Rather, by giving a broad but quick overview we hope to convey that spatial coupling is a fairly general method that can be used in a variety of areas and applications.





General one-dimensional systems

General Coupled One-Dimensional Analysis

Balance of areas in the EXIT chart of uncoupled ensembles gives the BP threshold of coupled systems

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The analysis presented in the previous part can be extended to general one-dimensional systems, i.e., systems where the “state” is a scalar and where the “action” can be described by two functions just like for the BEC. It can also be used as an approximation to higher (or infinite)-dimensional systems (like for BMS channels) in the spirit of the Gaussian approximation which is typically used for EXIT charts.

We present three examples of general one-dimensional systems. In the first example we consider transmission of an irregular ensemble over the BEC. From standard DE analysis, the BP threshold of the uncoupled ensemble is ≈ 0.3531. Applying the previous results on balance of areas in the EXIT chart method, we conclude that the BP threshold of the coupled system is given by ≈ 0.4032. From the Maxwell construction, this is also the MAP threshold of the uncoupled ensemble. The second example we consider is the transmission over the BAWGN channel. DE in this case consists of the evolution of general densities which, in general, cannot be represented by finite parameters. As a consequence, the analysis is hard. However, one classical approach towards analysis of such infinite dimension systems is to consider the Gaussian approximation (GA) of densities. Consequently, the DE is again a one-dimensional system. Although GA is not rigorous it gives an idea about the behavior of the system. Shown in the slides is the transmission of (3,6) regular ensemble over BAWGN. Applying GA and utilizing the EXIT chart method we see that the BP threshold is close to 0.42915. In fact, if we perform real density evolution we can determine tht the BP threshold is ≈ 0.4293, which is quite close. Applying the balance of areas result we get that the coupled ensemble has a threshold close to ≈ 0.4758. Again, density evolution on the coupled ensemble shows that the actual BP threshold of the coupled ensemble is close to ≈ 0.4794.

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Irregular Ensembles

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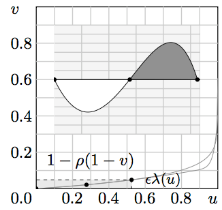

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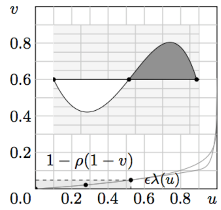

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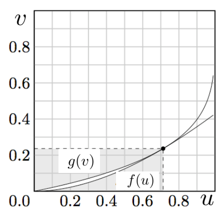

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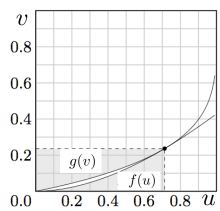

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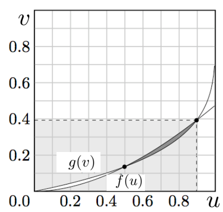

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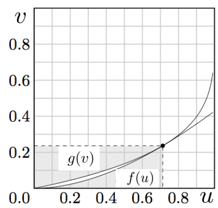

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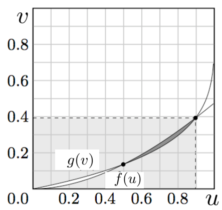

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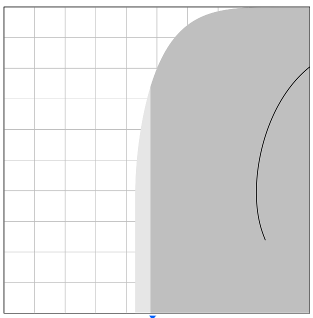
General BMS channels

Coupled Codes are Provably Capacity-Achieving under BP Decoding and they are universal with respect to all BMS channels

In the previous slide we saw that even general BMS channels can be analyzed using the EXIT chart method and the Gaussian approximation. Of course, this analysis is not exact, but it gives a quick and insightful idea about the performance. Let us now quickly discuss, how to exactly analyze spatially coupled ensembles for transmission over BMS channels.

General BMS Channels - Threshold Saturation

BAWGN Channel



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(3,6,L) coupled code ensemble with increasing L

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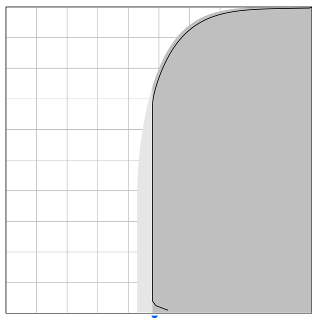
General BMS Channels - Threshold Saturation

BAWGN Channel



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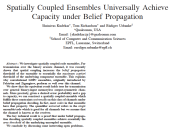
General BMS Channels - Main Statement

For a fixed spatially coupled ensemble with parameters (d_l, d_r, w, L) and a given BMS channel,

$$h^{\text{Area}} - O(1/\sqrt{w}) \leq h^{\text{BP}}_{\text{coupled}} \leq h^{\text{MAP}}_{\text{coupled}} \leq h^{\text{Area}} + O(w/L)$$

where the bounds are independent of the channel.


Spatially Coupled Ensembles Universally Achieve Capacity under Belief Propagation

In other words, we can construct codes which are universally capacity-achieving under BP decoding. The above statement provides the details. Here, h^{Area} denotes the area threshold of the uncoupled code ensemble, which in the case of general channels can be defined precisely to be equal to the channel entropy for which the area under the GEXIT curve is equal to the design rate. The theorem has been proven for the randomized ensemble. It states that the BP threshold and the MAP threshold of the coupled code ensemble are within O(1/sqrt(w)) of the area threshold of the underlying uncoupled ensemble. Furthermore, the area threshold can be shown to approach the Shannon threshold by increasing the constituent degrees. The O(1/sqrt(w)) is a very weak bound. The "true" behavior is conjectured to be exponentially small in w.

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(h^{Area},2006 Cyril Measson)

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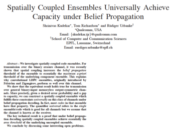
uniformly over all channels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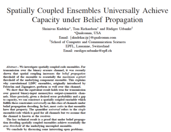
Spatially Coupled Ensembles Universally Achieve Capacity under Belief Propagation

Concentration: Almost all elements of the ensemble are good for all channels.

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Most Codes are Universal

Let $\mathcal{C}(c)$ denote the set of all BMS channels with capacity c *and let* $\epsilon > 0$. Then there exists a fixed spatially coupled code ensemble of rate at least $c - \epsilon$ such that *almost every* code in the ensemble is *good* for *all* channels in $\mathcal{C}(c)$.

Good means that we can transmit using ***belief propagation*** decoding with (block/bit) error probability at most ϵ.

This statement proves the *universality* of the coupled code ensemble. In other words, one coupled code ensemble can be used to transmit with rates arbitrarily close to the capacity of any channel with a given capacity, and achieve an arbitrarily small (block/bit) error rate while using low-complexity BP decoder. Furthermore, almost every code in the ensemble has this property. To prove this we use the fact the set of channel distributions with capacity at least R, is a compact set when we consider the Wasserstein metric. We then produce a finite cover of this set, such that every channel density lies within a distance δ of an element of the cover and furthermore the cover "dominates" (is degraded wrt) every channel density. To prove that the block error rate also goes to zero one can show that the spatially coupled codes, with variable node degrees at least 5, is an expander with sufficient expansion. Then one can use the BP decoder to bring down the bit error rate to a small value and show that by switching to the flipping decoder, one can correct any residual errors, thanks to the expansion.

General BMS Channels - Proofs of Threshold Saturation

Generalized EXIT Analysis:

arXiv:1201.2999 [pdf, ps, other]
Spatially Coupled Ensembles Universally Achieve Capacity under Belief Propagation
Shrinivas Kudekar, Tom Richardson, Ruediger Urbanke
Comments: 50 pages, 9 figures
Subjects: **Information Theory (cs.IT)**
http://arxiv.org/pdf/1201.2999.pdf

Potential Function Analysis:

arXiv:1301.6111 [pdf, ps, other]
A Proof of Threshold Saturation for Spatially-Coupled LDPC Codes on BMS Channels
Santhosh Kumar, Andrew J. Young, Nicolas Macris, Henry D. Pfister
Comments: In proceedings of Allerton 2012; Corrected a typo in equation (5)
Subjects: **Information Theory (cs.IT)**
http://arxiv.org/pdf/1301.6111.pdf

The original proof for the threshold saturation phenomena was furnished in “Spatially Coupled Ensembles Universally Achieve Capacity under Belief Propagation”, Kudekar, Richardson and Urbanke ’12. In this article, it is also shown that the spatially coupled ensembles universally achieve capacity under BP decoding. The proof technique involved demonstrating the existence of a special FP of DE of the coupled ensembles. This special FP, as seen in the proof for the case of BEC, has a long tail of densities which are almost perfectly decoder (i.e., one can imagine that the associated probability of error is very close to zero), a quick transition and then a large flat part where densities are equal to the forward FP of DE for the underlying uncoupled code ensemble. It is then shown that this special FP, if it exists, can only do so at the a channel entropy close to the area threshold of the underlying uncoupled ensemble. More precisely, the channel entropy must be within O(1/sqrt(w)) of the area threshold. This is shown using the generalized EXIT function.Then, it is shown that for a channel with entropy strictly less than the area threshold minus the wiggle O(1/sqrt(w)), the forward FP of DE (i.e., the density under BP decoding) must converge to a trivial FP, i.e., perfect decoding. Because, if it did not, then DE must be stuck in an FP which is “special” as mentioned above. But then any such special FP can not have a channel entropy value more than O(1/sqrt(w)) away from the area threshold. More recently, Kumar, Young, Macris, Pfister have furnished another proof of the threshold saturation phenomena using the potential function approach mentioned previously. The proof again involves constructing an appropriate potential function, which closely resembles the replica symmetric free energy of the system.

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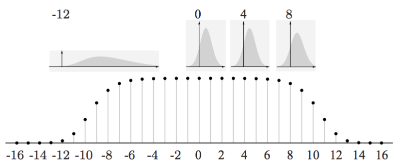

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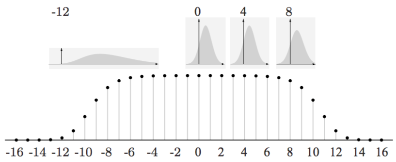

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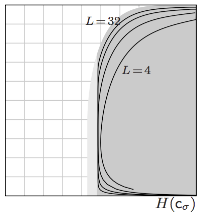

(generalized) EXIT

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Multi-user Communications Finite-state channels, Noisy Slepian-Wolf

Recently, it was also shown that spatially coupled codes are capacity-achieving and universal in host of other communication scenarios. The list is not exhaustive and we will mention only a few examples.

Multi-Access Channels

Gaussian MAC Channel

$$Y = h_1 X_1 + h_2 X_2 + N$$

arXiv:1102.2856 [pdf, other]
Spatially Coupled Codes over the Multiple Access Channel
Shrinivas Kudekar, Kenta Kasai
Subjects: Information Theory (cs.IT)

Consider transmission over a two user multi-access channel with AWGN noise. Considers a fading channel, with different fades, for the two users. More precisely, consider slow fading channels, i.e., the channel gains are unknown but fixed. Consider the subset of the region of fading coefficients for which reliable transmission is possible under a fixed rate pair. Above, the pentagon in red is the achievable region under MAP decoding. It is observed that when both the users use a standard (3,6) LDPC code ensemble to transmit, the achievable region is much smaller than the optimal one. In fact, even if one uses an ensemble optimized for equal received power case (i.e, $h_1 = h_2$), it does not cover the entire achievable region. However, it is shown that if both users use coupled code ensemble of increasing degrees, then the achievable region, under BP decoding, approaches the optimal region. Thus the threshold saturation phenomena is also manifested in this case.

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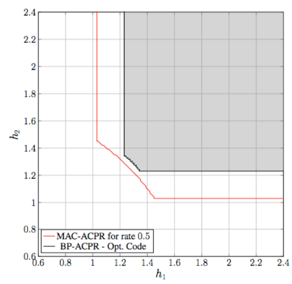

[Figures from Yedla et al.]

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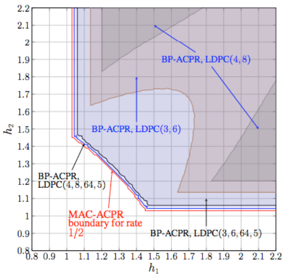

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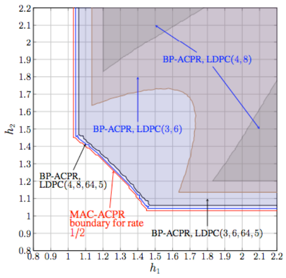

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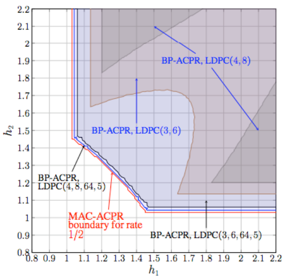

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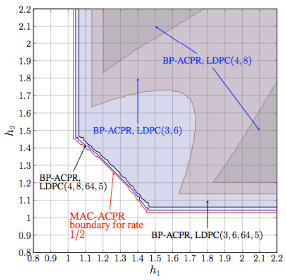

[Figures from Yedla et al.]

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arXiv:1110.0252 [pdf, other]
Universal Codes for the Gaussian MAC via Spatial Coupling
Arvind Yedla, Phong S. Nguyen, Henry D. Pfister, Krishna R. Narayanan
Comments: 8 pages, to appear in proceedings of Allerton 2011
Subjects: **Information Theory (cs.IT)**

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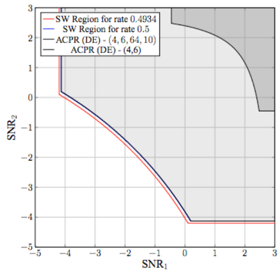
Noisy Slepian-Wolf

Correlated sources (BSC) transmitted over BAWGNC(h)

Symmetric channel conditions, joint iterative decoding

[Figure from Yedla et al.]



Saturday, July 13, 13

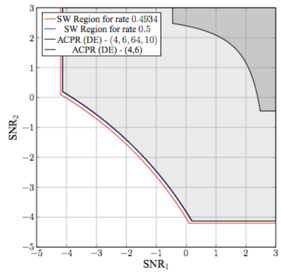
In this case it is shown that the threshold saturation phenomena also occurs when transmitting two correlated sources over noisy channels. Consider two sources correlated via a virtual BSC(p) channel. I.e., imagine a source U1 which is Bern(1/2) and consider another source U2 which is obtained from U1 by transmitting it over BSC(p). The two sources are then independently coded and then transmitted on two AWGN channels with noise variances equal to 1/SNR1 and 1/SNR2. It is assumed that both the sources use the same code ensemble, and thus the same rate, to communicate over the noisy channel. The receiver has the knowledge of both the source correlation and the channel parameters. Shown in the figure is the Slepian-Wolf (capacity) achievable region for this problem. It is desirable to construct a code such that one is able to transmit at all possible channel value pairs (for a given rate pair) in the achievable region. This would ensure that the scheme is universal, i.e., attains near-capacity performance without channel knowledge at the transmitter. As shown in the figure, if one uses a standard (4,6) code to transmit at (rate1, rate2) = (1/3, 1/3) using BP decoding, then the achievable region is considerably smaller than the Slepian-Wolf region. Note that here the virtual correlation channel is BSC(p=0.11). Hence, the minimum rate at which each source can transmit is equal to h_2(p) = 1/2. However, using randomized coupled code ensemble (4,6,64,10) we observe that near-capacity performance is achievable. Note that using the coupled code results in a slight rate-loss. This is reflected in the figure by the Slepian-Wolf region for rate 0.4934.

Noisy Slepian-Wolf

Correlated sources (BSC) transmitted over BAWGNC(h)

Symmetric channel conditions, joint iterative decoding

[Figure from Yedla et al.]



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arXiv:1105.6374 [pdf, ps, other]
Universality for the Noisy Slepian-Wolf Problem Via Spatial Coupling
Arvind Yedla, Henry D. Pfister, Krishna Narayanan
Comments: 5 pages, 8 figures, to appear in ISIT 2011
Subjects: **Information Theory (cs.IT)**

Saturday, July 13, 13 14

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Finite-State Channels

Generalized Erasure Channel

Output of a binary input linear filter (1-D) transmitted over erasure channel

arXiv:1102.0406 [pdf, ps, other]
Threshold Saturation on Channels with Memory via Spatial Coupling
Shrinivas Kudekar, Kenta Kasai
Comments: Submitted to ISIT 2011
Subjects: **Information Theory (cs.IT)**

Saturday, July 13, 13

In this case one considers transmission over a channel with memory. We consider the simplest case of a memory 2 channel with erasure. More precisely, we have the output of a linear filter Y_i = X_{i} - X_{i-1} which is then transmitted over an erasure channel. It is observed, by plotting the corresponding EXIT curves, that the symmetric information capacity is achieved by considering spatially coupled codes. This phenomena also extends to the dicode AWGN channel, where in we transmit the output of the linear filter over an AWGN channel.

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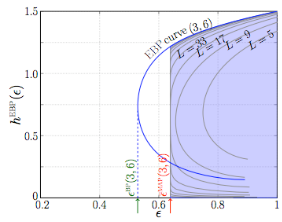

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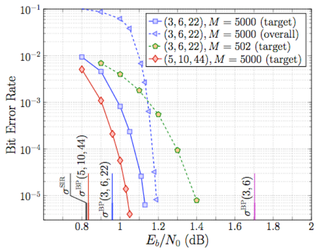

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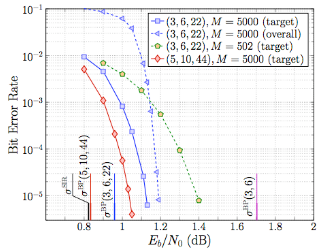

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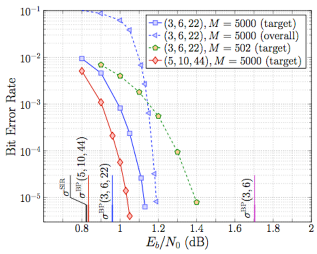

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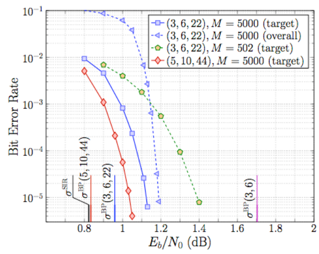

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arXiv:1107.3253 [pdf, other]
Spatially-Coupled Codes and Threshold Saturation on Intersymbol-Interference Channels
Phong S. Nguyen, Arvind Yedla, Henry D. Pfister, Krishna R. Narayanan
Comments: 30 pages, 10 figures
Subjects: **Information Theory (cs.IT)**

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Many more...

Trapping set, Pseudocodeword analysis

arXiv:1106.1414 [pdf, ps, other]
Exact Free Distance and Trapping Set Growth Rates for LDPC Convolutional Codes
David G. M. Mitchell, Ali E. Pusane, Michael Lentmaier, Daniel J. Costello Jr

arXiv:1102.3936 [pdf, ps, other]
AWGN Channel Analysis of Terminated LDPC Convolutional Codes
David. G. M. Mitchell, Michael Lentmaier, Daniel J. Costello, Jr

arXiv:1004.5157 [pdf, ps, other]
Deriving Good LDPC Convolutional Codes from LDPC Block Codes
Ali E. Pusane, Roxana Smarandache, Pascal O. Vontobel, Daniel J. Costello Jr

Weight Distribution Minimum Distance

arXiv:1104.0599 [pdf, ps, other]
Near concavity of the growth rate for coupled LDPC chains
S. Hamed Hassani, Nicolas Macris, Ryuhei Mori

arXiv:1303.1858 [pdf, ps, other]
On the Minimum Distance of Generalized Spatially Coupled LDPC Codes
David G. M. Mitchell, Michael Lentmaier, Daniel J. Costello Jr

CDMA

arXiv:1206.5919 [pdf, ps, other]
Performance Improvement of Iterative Multiuser Detection for Large Sparsely-Spread CDMA Systems by Spatial Coupling
Keigo Takeuchi, Toshiyuki Tanaka, Tsutomu Kawabata

arXiv:1209.5785 [pdf, other]
Coupling Data Transmission for Capacity-Achieving Multiple-Access Communications
Dmitri Truhachev, Christian Schlegel

arXiv:1205.3317 [pdf, ps, other]
Spatially-Coupled Random Access on Graphs
Gianluigi Liva, Enrico Paolini, Michael Lentmaier, Marco Chiani

Relay Channel

arXiv:1102.5087 [pdf, ps, other]
Spatially Coupled LDPC Codes for Decode-and-Forward in Erasure Relay Channel
Hironori Uchikawa, Kenta Kasai, Kohichi Sakaniwa

The spatial coupling principle has been applied to many other fundamental problems. We list here only a handful of them which show the manifestation of the threshold saturation phenomena in different problems. We apologize for any papers we have left out.

Many more...

Wiretap Channel

arXiv:1010.1669 [pdf, ps, other]
Rate-Equivocation Optimal Spatially Coupled LDPC Codes for the BEC Wiretap Channel
Vishwambhar Rathi, Ruediger Urbanke, Mattias Andersson, Mikael Skoglund

Rateless Coding

arXiv:1108.0535 [pdf, other]
Universal Rateless Codes From Coupled LT Codes
Vahid Aref, Rüdiger L. Urbanke

arXiv:1302.1511 [pdf, ps, other]
Spatially-Coupled Precoded Rateless Codes
Kosuke Sakata, Kenta Kasai, Kohichi Sakaniwa

Lattice codes

arXiv:1107.4900 [pdf, ps, other]
Threshold Improvement of Low-Density Lattice Codes via Spatial Coupling
Hironori Uchikawa, Brian M. Kurkoski, Kenta Kasai, Kohichi Sakaniwa

Lossy Source Coding

arXiv:1202.4959 [pdf, ps, other]
Lossy Source Coding via Spatially Coupled LDGM Ensembles
Vahid Aref, Nicolas Macris, Rudiger Urbanke, Marc Vuffray

Bounded Complexity

arXiv:1102.4612 [pdf, ps, other]
Spatially-Coupled MacKay-Neal Codes and Hsu-Anastasopoulos Codes
Kenta Kasai, Kohichi Sakaniwa

Quantum Error Correction

arXiv:1102.3181 [pdf, ps, other]
Spatially Coupled Quasi-Cyclic Quantum LDPC Codes
Manabu Hagiwara, Kenta Kasai, Hideki Imai, Kohichi Sakaniwa

I. Andriyanova, D. Maurice, J.-P. Tillich, *Quantum spatially-coupled LDGM codes*

Linear Programming Decoding

arXiv:1301.6410 [pdf, ps, other]
Linear Programming Decoding of Spatially Coupled Codes
Louay Bazzi, Badih Ghazi, Rudiger Urbanke

The spatial coupling principle has been applied to many other fundamental problems. We list here only a handful of them which show the manifestation of the threshold saturation phenomena in different problems. We apologize for not able to list all of them due to space constraints.

Problems beyond Communications - Compressive Sensing and K-Satisfiability

Compressive Sensing

$$y = Ax + w$$

In this case we consider the problem of compressive sensing. This is a classical problem in signal processing and deals with the situation when there is a system of noisy linear measurements where the number of measurements is less than the ambient dimension of the signal which is measured. Of course it is not possible to recover the original signal in general in such a case, but if we impose some suitable structure on the signal it becomes feasible. The most common constraint on the signal is that it is **sparse** in some basis. Above, A denotes the measurement matrix, x the ambient signal which is sparse, with sparsity k < n, and y is the measurement vector of length m. Further, w is the AWG noise. Clearly, for a recovery of any sort, m >= k. We consider the regime where m, n, k go to infinity with m/n = delta and k/n = epsilon both constant. In this case, one is interested to known the phase transition or the tradeoff between delta (undersampling ratio) and the sparsity (epsilon). It is clear that the larger the sparsity, the larger is the required number of measurements for robust recovery. Here, robust means that the noise which is present in the observation enters the estimate only in a bounded fashion.

Compressive Sensing

$$y = Ax + w$$

$$\begin{pmatrix} y \end{pmatrix}_{m \times 1} = \begin{pmatrix} \quad A \quad \end{pmatrix}_{m \times n} \begin{pmatrix} x \end{pmatrix}_{n \times 1} + \begin{pmatrix} w \end{pmatrix}_{m \times 1}$$

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$m < n$ $\quad x$ is sparse $\quad f_X(x)$ cont. dist. with $1 - \epsilon$ mass at zero

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Regime: $m, n \to \infty \quad \delta = m/n \quad \epsilon = k/n$

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(Bayesian) Compressive Sensing

Traditionally compressive sensing deals with the design and analysis when one wants to recover **all** signals with a given sparsity. This is a strong recovery condition and one can consider a slightly weaker condition where one considers a prior distribution on the signal with sparsity constraint of epsilon. It was shown recently by Wu and Verdu, that the minimum number of measurements, under optimal decoding, required to robustly recover a signal with distribution f_X(x) is equal to n*d(f_X(x)), where d(f_X(x)) is the information dimension associated to the distribution d(f_X(x)). For the case of sparse signals with sparsity eps, d(f_X(x)) <= eps. Thus the optimal sparsity-undersampling tradeoff is given by a 45º line. However, this is achieved under optimal decoding and a priori this is computationally expensive.

(Bayesian) Compressive Sensing

$$m \geq nd(f_X) + o(n)$$

$d(f_X)$ is the Renyi Information Dimension

Rényi information dimension: Fundamental limits of almost lossless analog compression
Y Wu, S Verdú - **Information** Theory, IEEE Transactions on, 2010 - ieeexplore.ieee.org

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$d(f_X)$ is less than ϵ for signal with sparsity equal to ϵ

Rényi information dimension: Fundamental limits of almost lossless analog compression
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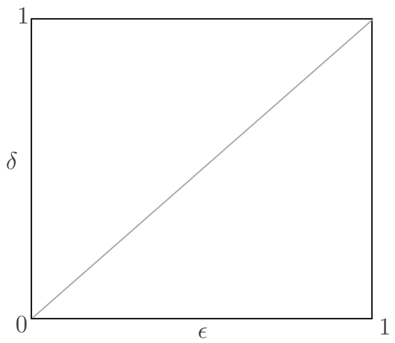

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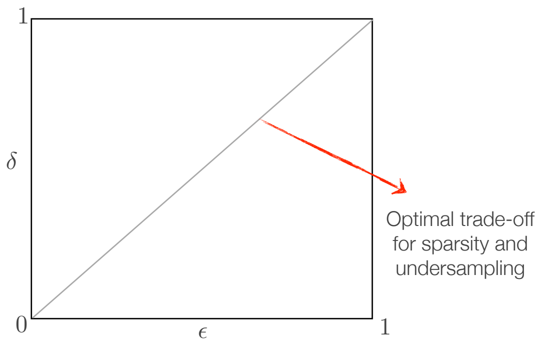

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Compressive Sensing and Coupling

[Kudekar, Pfister, '10] [Krzakala, Mezard, Sausset, Sun, Zdeborova, '12] [Donoho, Javanmard, Montanari, '12]

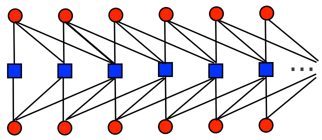
Using coupled measurement matrices with message-passing decoder

Recently, it was shown that in fact spatial coupling can be used to design measurement matrices which can achieve the optimal undersampling-sparsity tradeoff using a low-complexity message-passing decoder. The basic idea is very similar to the coding case. In coding, we have at the boundary of the code additional knowledge. This knowledge makes it easier to decode bits close to the boundary. This effect then propagates along the chain of the code through the coupled structure. In the papers by Krzakala et al. and Donoho et al., the authors construct a measurement matrix ensemble which is "lifted" from a base matrix as shown in the slides. This base matrix has the property that at the boundary there are more measurements. I.e., one can have an undersampling ratio which is much larger than the target one. However, these are small compared to the total measurements and thus asymptotically the undersampling ratio is not affected. Now the large number of measurements at the boundary help "kickstart" the decoding process even when we are very close to the optimal delta-eps tradeoff curve. Then the coupling structure again helps to decode the rest of the signal.

Compressive Sensing and Coupling

[Kudekar, Pfister, '10] [Krzakala, Mezard, Sausset, Sun, Zdeborova, '12] [Donoho, Javanmard, Montanari, '12]

Using coupled measurement matrices with message-passing decoder



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Using coupled measurement matrices with message-passing decoder

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & \dots & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix}$$

Band Diagonal Adjacency Matrix
(Base) Measurement Matrix

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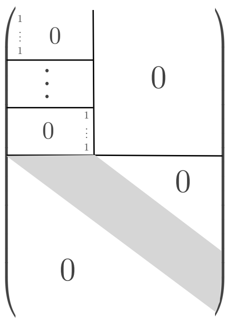
base measurement matrix representing heteroscedasticity



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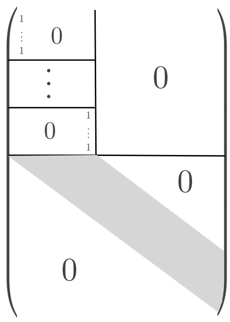

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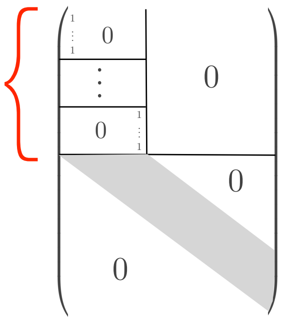
does not affect overall undersampling



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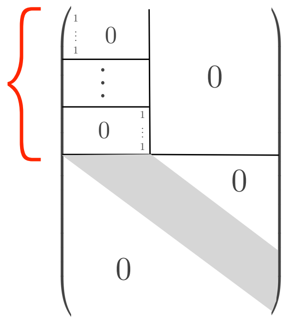

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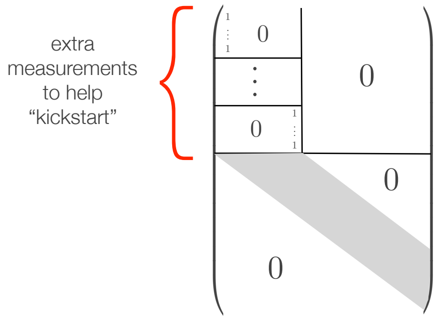

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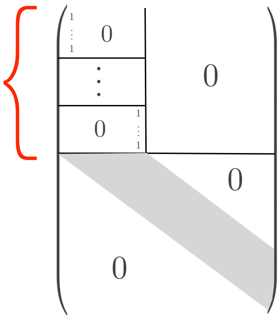

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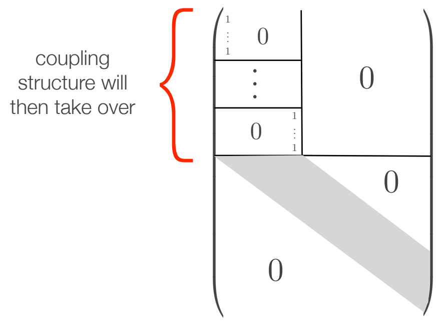

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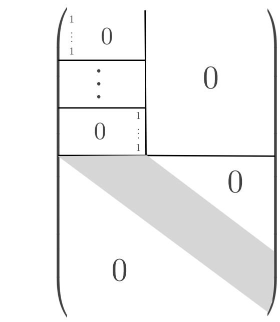

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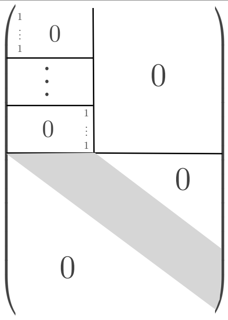

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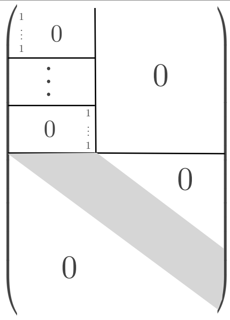

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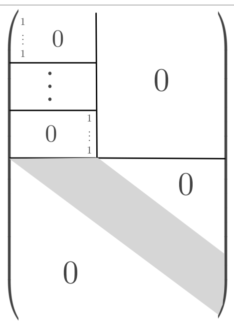

$+$ Approximate Message Passing (AMP) decoder

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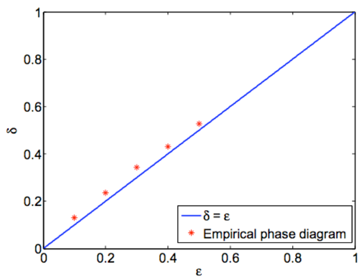

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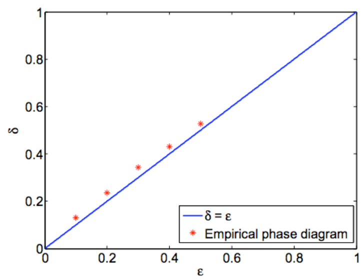

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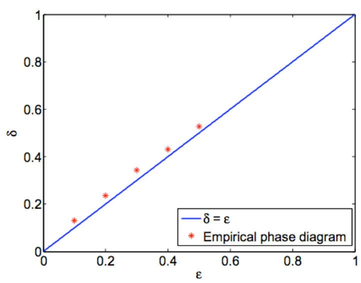

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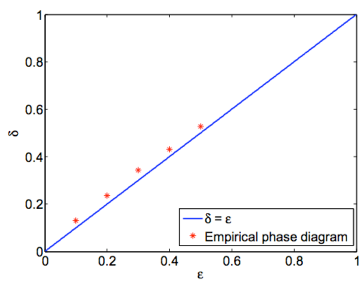

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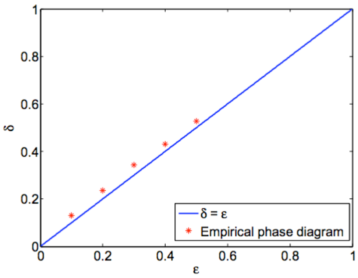

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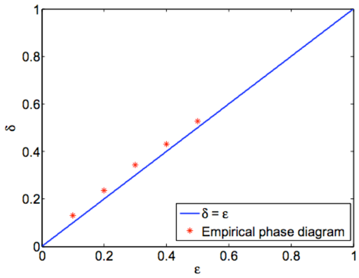

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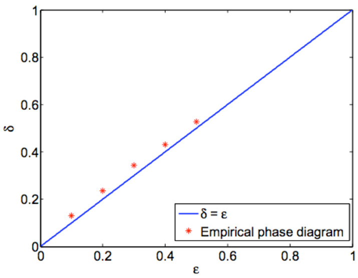
Compressive Sensing and Coupling

[Kudekar, Pfister, '10] [Krzakala, Mezard, Sausset, Sun, Zdeborova, '12] [Donoho, Javanmard, Montanari, '12]



[Figure from Javanmard et al]

References

Donoho, D. L., Javanmard, A., & Montanari, A. (2012, July). Information-theoretically optimal compressed sensing via spatial coupling and approximate message passing. In *Information Theory Proceedings (ISIT), 2012 IEEE International Symposium on* (pp. 1231-1235). IEEE.

Recently, it was shown that in fact spatial coupling can be used to design measurement matrices which can achieve the optimal undersampling-sparsity tradeoff using a low-complexity message-passing decoder. The basic idea is very similar to the coding case. In coding, we have at the boundary of the code additional knowledge. This knowledge makes it easier to decode bits close to the boundary. This effect then propagates along the chain of the code through the coupled structure. In the papers by Krzakala et al. and Donoho et al., the authors construct a measurement matrix ensemble which is "lifted" from a base matrix as shown in the slides. This base matrix has the property that at the boundary there are more measurements. I.e., one can have an undersampling ratio which is much larger than the target one. However, these are small compared to the total measurements and thus asymptotically the undersampling ratio is not affected. Now the large number of measurements at the boundary help "kickstart" the decoding process even when we are very close to the optimal delta-eps tradeoff curve. Then the coupling structure again helps to decode the rest of the signal.

Compressive Sensing - Proof of Threshold Saturation

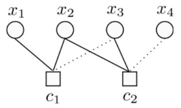
11. arXiv:1112.0708 [pdf, other]
Information-Theoretically Optimal Compressed Sensing via Spatial Coupling and Approximate Message Passing
David L. Donoho, Adel Javanmard, Andrea Montanari

Proof based on:

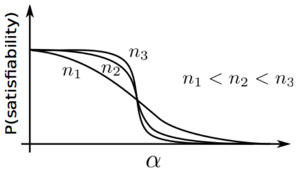
State evolution (evolution of the MSE under AMP) + Continuum analysis + Potential function analysis

A proof of the threshold saturation phenomena was provided recently. This proof independently developed the continuum and the potential function analysis to prove the threshold saturation phenomena for the compressive sensing problem.

K-Satisfiability -- Setup



$$\lim_{n\to\infty} \mathbb{P}\left[\mathcal{F}(n, K, M = \alpha n) \text{ is satisfied}\right] = \begin{cases} 0, & \alpha > \alpha_K, \\ 1, & \alpha < \alpha_K. \end{cases}$$



Suppose that we are given a set of n Boolean variables $\{x_1, \ldots, x_n\}$. Each variable x_i can take on the values 0 and 1, where 0 means "false" and 1 means "true". We define a literal to be either a variable xi or its negation $\bar{x}_i$. A clause is a disjunction of literals, e.g., $C = x_1 \vee x_2 \vee \bar{x}_3$ where the operator "$\vee$" denotes the Boolean "or" operator. An assignment is an assignment of values to the Boolean variables, e.g., $x_1 = 0$, $x_2 = 1$, and $x_3 = 0$. Such an assignment will either make a clause satisfy or not satisfy. For example the clause $x_1 \vee x_2 \vee \bar{x}_3$ with assignment $x_1 = 0$, $x_2 = 1$, and $x_3 = 0$ evaluates to 1 which is satisfied. A SAT formula is a conjunction of a set of clauses. For example, F which is defined as $F=(x_1 \vee x_2 \vee \bar{x}_3)\wedge(x_2 \vee \bar{x}_4)\wedge x_3$ is a SAT formula. Given a SAT formula F, we associate to it a bipartite graph G. The vertices of the graph are $V \cup C$, where $V = \{x_1,\ldots,x_n\}$ are the Boolean variables and $C = \{c_1,\ldots,c_M\}$ are the M clauses. There is an edge between x_i and c_j if and only if x_i or $\bar{x}_i$ is contained in the clause c_j. Further we draw a "solid line" if c_j contains x_i and a "dashed line" if c_j contains $\bar{x}_i$. In the slide above such a factor graph is shown. We talk about a K-SAT formula if each clause contains exactly K Boolean variables and we talk about random K-SAT formulas if we pick formulas from an ensemble. We define the ensembles of formulas, call it F(n,K,M), by showing how to sample from it. To this end, pick M clauses independently, where each clause is chosen uniformly at random from the n choose k times 2^k possible clauses. Then form F as the conjunction of these M clauses. Now let us consider the following experiment. Fix $K \geq 3$ (e.g., $K = 3$) and sample from the F(n,K,M) ensemble. Is such a formula satisfiable with high probability? It turns out that the most important parameter that effects the answer is $\alpha = M/n$.

K-Satisfiability -- Effect of Spatial Coupling

As for coding we can construct spatially coupled K-SAT formulas and we can show that for many algorithms the threshold of M/n up to which one can find satisfiable assignments is improved.

Combining this with the interpolation technique this can be used to prove better lower bounds on the SAT/UNSAT thresholds of uncoupled formulas.

arXiv:1112.6320 [pdf, ps, other]
Threshold Saturation in Spatially Coupled Constraint Satisfaction Problems
S. Hamed Hassani, Nicolas Macris, Rudiger Urbanke
Subjects: **Computational Complexity (cs.CC)**; Statistical Mechanics (cond-mat.stat-mech); Information Theory (cs.IT)

Some more ...

arXiv:1105.0807 [pdf, ps, other]
Chains of Mean Field Models
S. Hamed Hassani, Nicolas Macris, Ruediger Urbanke
Subjects: **Discrete Mathematics (cs.DM)**; Statistical Mechanics (cond-mat.stat-mech); Information Theory (cs.IT)

arXiv:1102.3056 [pdf, ps, other]
A Phenomenological Study on Threshold Improvement via Spatial Coupling
Keigo Takeuchi, Toshiyuki Tanaka, Tsutomu Kawabata
Comments: re-submitted to IEICE Trans. Fundamentals
Subjects: **Information Theory (cs.IT)**

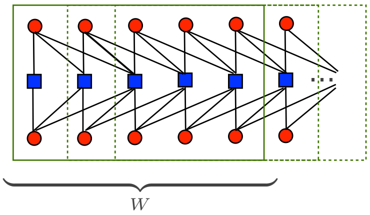
arXiv:1105.0785 [pdf, ps, other]
Coupled Graphical Models and Their Thresholds
S. Hamed Hassani, Nicolas Macris, Ruediger Urbanke
Comments: In proceedings of ITW 2010
Subjects: **Information Theory (cs.IT)**; Statistical Mechanics (cond-mat.stat-mech); Discrete Mathematics (cs.DM)

arXiv:1303.0540 [pdf, ps, other]
The Space of Solutions of Coupled XORSAT Formulae
S. Hamed Hassani, Nicolas Macris, Rudiger Urbanke
Comments: Submitted to ISIT 2013
Subjects: **Disordered Systems and Neural Networks (cond-mat.dis-nn)**; Discrete Mathematics (cs.DM); Information Theory (cs.IT)

Part III: Practical Aspects and Open Questions

Windowed Decoding

(d_l, d_r, L, W) W is the size of the sliding window



As shown previously, to achieve the capacity, coupled codes must have longer and longer chains. This implies large blocklengths which in turn implies large latency and decoding complexity. In order to retain the attractive performance of spatially coupled codes and have low latency and decoding complexity at the same time, it was proposed in "Windowed Decoding of Spatially Coupled Codes", Iyengar, Siegel, Urbanke, Wolf, to use a windowed decoder. In this scheme, decoding is only carried out within a window that covers a portion of the chain smaller than the total length. Once the probability of error in that window has been brought down to the desired level, the window is shifted one section of the coupled code to the right and the decoding is performed again. It is also shown that the threshold of the windowed decoder, now defined as the channel value below which one can attain a target error rate, approaches exponentially fast in the window size to the threshold of the traditional BP decoder. In the waterfall region, the traditional BP complexity scales as O(ML^2), where recall that M is the size of the "lift". For windowed decoder of size W, the complexity is O(MW^{2}L). Thus for W < sqrt(L), the complexity is lower for the windowed decoder. Also, once the error rate in a window is brought down to the desired level, the decoder could output the target symbols. Thus the latency is reduced to W/L fraction of the BP decoder latency. We remark that sliding windowed decoder was essentially introduced in the original paper by Felstrom and Zigangirov, '99. It was called there as "pipe-lined" decoding. This was further analyzed in the paper by Lentmaier et al.

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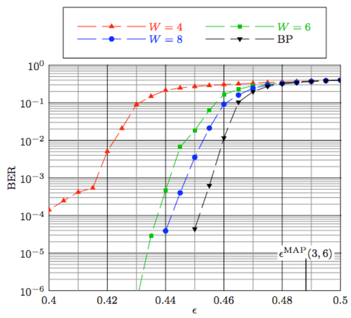

[Figure gratefully reproduced from Iyengar et al]

(3,6,64)

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Windowed Decoding

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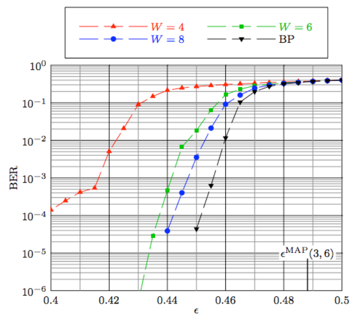
Complexity in the waterfall region scales like O(MW^{2}L) as opposed to O(ML^2)



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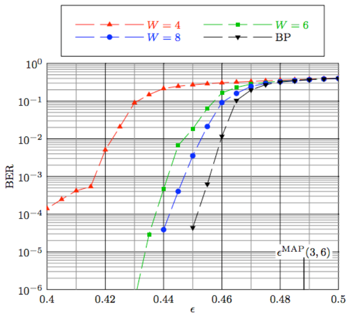

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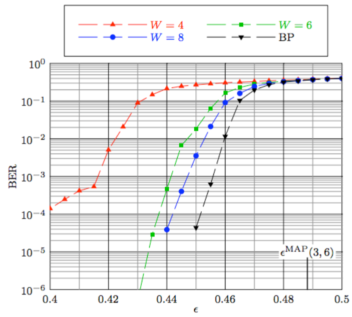

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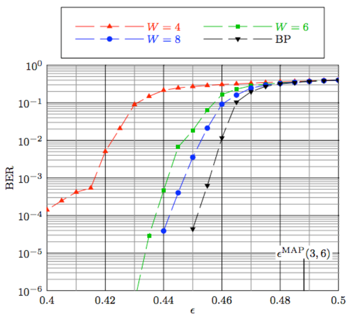

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(3,6,64)

Reference

A. J. Felström and K. S. Zigangirov, “Time-varying periodic convolutional codes with low-density parity-check matrix,” *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 2181–2190, Sept. 1999.

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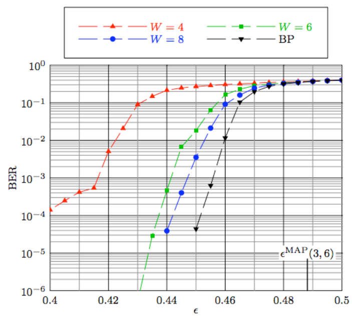

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Reference

M. Lentmaier, A. Sridharan, K. S. Zigangirov, and D. J. Costello, Jr., “Iterative decoding threshold analysis for LDPC convolutional codes,” *IEEE Trans. Info. Theory*, Oct. 2010. Vol. 56, No. 10, pp. 5274

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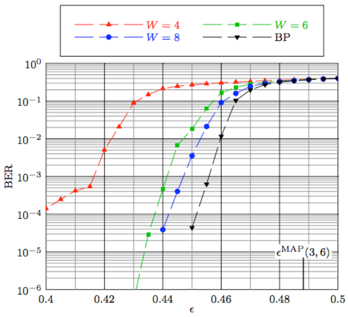

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Reference

arXiv:1106.0075 [pdf, other]
Windowed Decoding of Spatially Coupled Codes
Aravind R. Iyengar, Paul H. Siegel, Rudiger L. Urbanke, Jack K. Wolf
Comments: Accepted for publication in the IEEE Transactions on Information Theory, November 2012.

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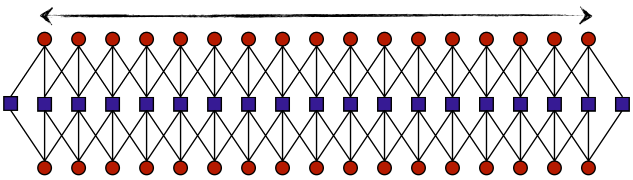
Rate-loss

Rate Loss Due to Boundary

d_l=3 d_r=6

L



One of the main reasons for the remarkable performance of the spatially coupled ensemble is the boundary effect. At the boundary, one has better error protection via smaller degree check nodes. This, however, also introduces a rate-loss. If one desires to construct a code for a particular target rate, the boundary causes the overall rate of the code to be slightly smaller. This is quantified in the slide above. We have also seen that if there is no termination, then the threshold does not saturate. It is thus desirable to saturate the threshold and at the same time reduce the rate-loss. Notice that as the length of the chain increases, the rate-loss can be made arbitrarily small.

Rate Loss Due to Boundary

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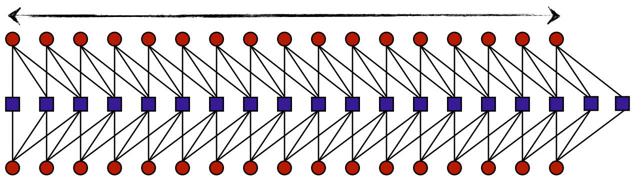

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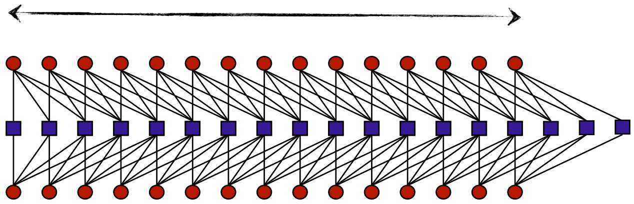
Rate Loss Due to Boundary

d_l=4 d_r=8

L



In this slide we see an example of rate-loss from coupling (dl, dr) regular LDPC code ensemble. The rate is dl(dr-1)/(L dr) less than the design rate. It is clear that the rate-loss goes to zero as L becomes large. However, increasing L causes the blocklength to increase. Hence, in practical systems it would be desirable to reduce this rate-loss.

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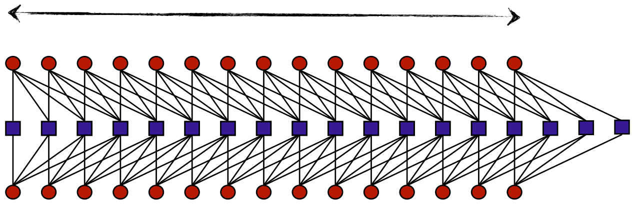

$\frac{d_r}{d_l}L$ variable nodes

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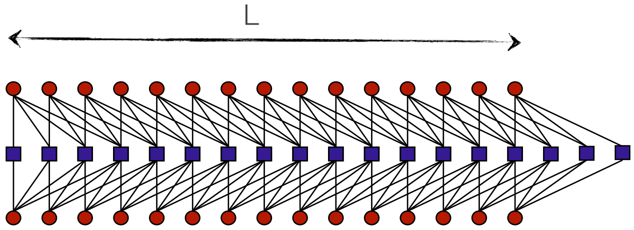

$\frac{d_r}{d_l}L$ variable nodes

$L + d_l - 1$ check nodes

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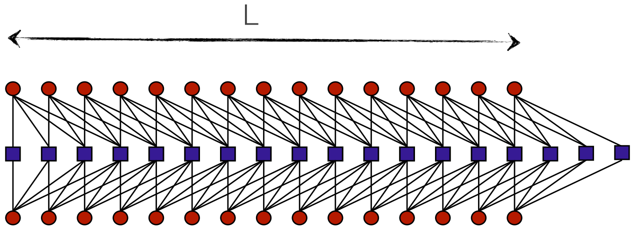

$\frac{d_r}{d_l}L$ variable nodes

$$r = 1 - \frac{d_l}{d_r} - \frac{d_l(d_l-1)}{d_r}\frac{1}{L}$$

$L + d_l - 1$ check nodes

In this slide we see an example of rate-loss from coupling (dl, dr) regular LDPC code ensemble. The rate is dl(dr-1)/(L dr) less than the design rate. It is clear that the rate-loss goes to zero as L becomes large. However, increasing L causes the blocklength to increase. Hence, in practical systems it would be desirable to reduce this rate-loss.

Rate Loss Due to Boundary

Can we do better?

There are several ways to reduce the rate-loss. It is also an interesting open question as to what the fundamental limits are for the rate-loss. We will present few techniques to reduce the rate-loss. Several such rate-loss mitigation techniques are presented in the article, “Threshold Saturation on BMS Channels via Spatial Coupling”, Kudekar, Richardson, Urbanke, 2010. One technique which we will not mention here is to think of the circular ensemble, wherein instead of the chain we have coupled codes arranged in a circle. It is not hard to see that the original coupled code along a chain is obtained by setting the appropriate consecutive bits in the circular ensemble to be known. This is equivalent to transmitting these bits over a BEC(0). Instead, we consider a scheme in which the “boundary” bits are not transmitted over BEC(0), but over some BEC(e) where e is close to zero. As a result we reduce the rate-loss. It is shown that even if we do not set the boundary bits to be perfectly known, the “wave” is still generated, and the threshold still saturates. Of course, this can be done only for e < e*, above which there is degradation in the threshold.

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Can we do better?

- Mitigating rate-loss
  - One-Sided Termination

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- Mitigating rate-loss
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  - Deletion

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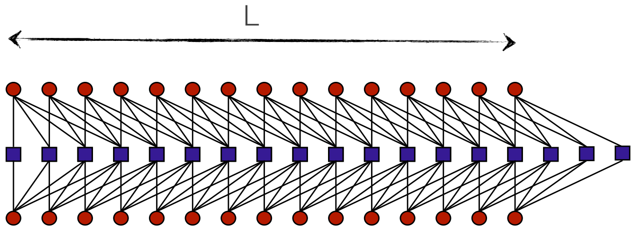
Mitigating rate-loss: One-sided termination

[Kudekar, Richardson, Urbanke, 2012]

$d_l=4$ $d_r=8$



It is observed that termination is not needed on both the sides of the coupled code. Termination or the boundary is only required at one side. As a consequence, the check nodes at the, say, right boundary can be combined to reduce the number of check nodes. Note that in this process, the degrees of the resultant check nodes increases. It is not hard to see that this leads to an immediate reduction of the rate-loss by half as is seen in the example above.

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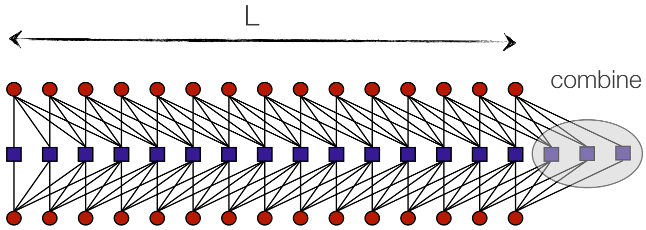

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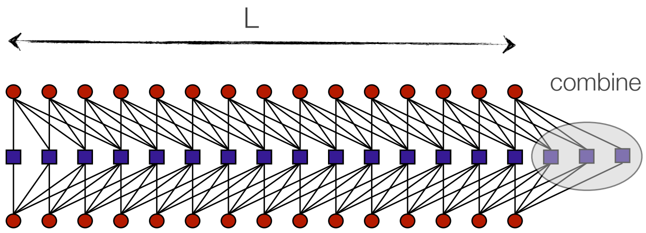

$\frac{d_l}{d_r}L$ variable nodes

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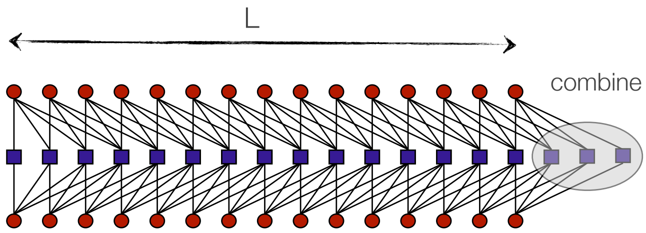

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$L+\frac{d_l-1}{2}$ check nodes

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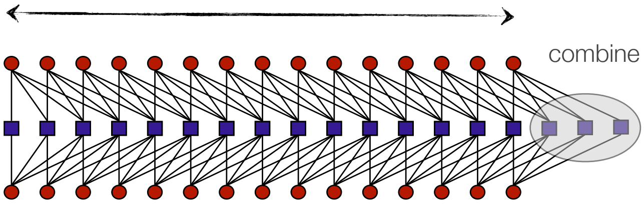

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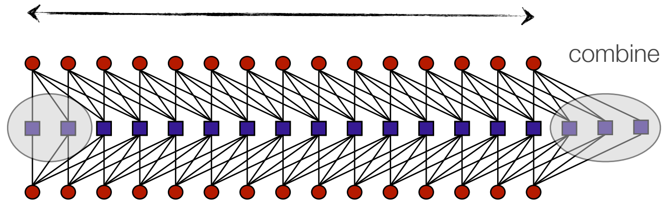

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Mitigation of rate-loss: Deletion

[Kasai et. al. ITW 2012]

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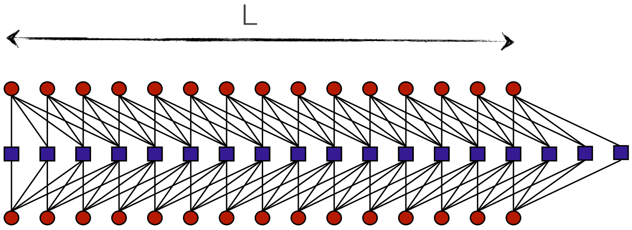
L



Rather than merging the check nodes at the right boundary, one can delete the check nodes. This reduces the number of check nodes and again reduces the rate-loss. Note that with the deletion of the check nodes we introduce variable nodes of lesser degrees. However, it is still observed, (see “Efficient Termination of Spatially-Coupled Codes”, Tazoe, Kasai and Sakaniwa, 2012) that the “wave” which begins at the left boundary travels all the way through to the right. A nice consequence of this technique is that the rate-loss is independent of the degrees. As shown in the slide, the rate loss just depends on the ratio dl/dr and not on the constituent degrees as the previous method did.

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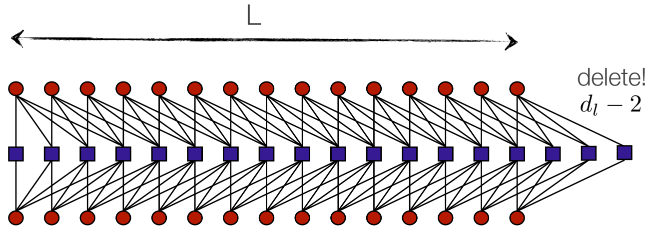

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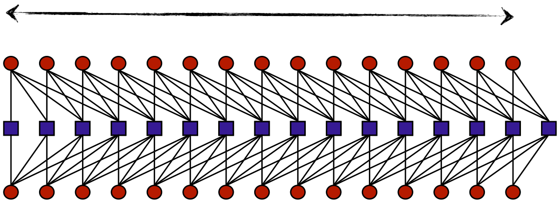

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$d_l - 2$

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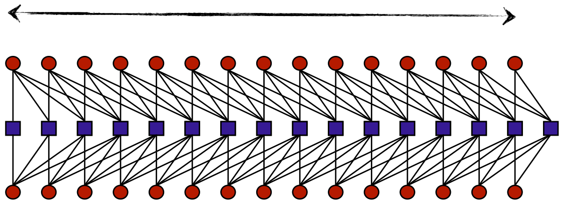

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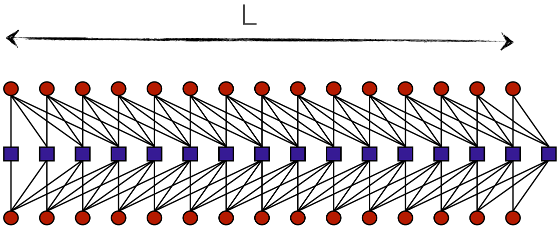

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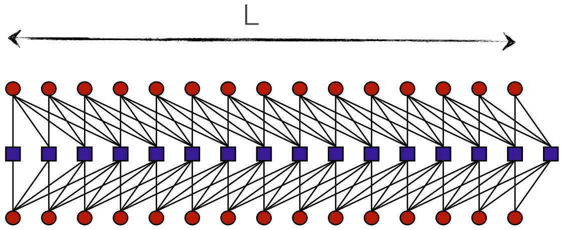

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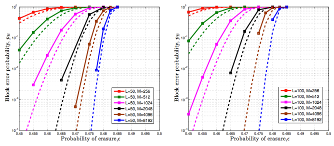
Finite-length Scaling

Finite-Length Scaling (BEC)

$$P_B \sim L\sqrt{M}\delta e^{-\kappa M \delta^2} \qquad n = LM \qquad \Delta R = \frac{\beta}{L}$$



$$\text{gap to capacity} = \delta + \frac{\beta}{L}$$

$$= \frac{\alpha}{\sqrt{n}}\sqrt{L} + \frac{\beta}{L}$$

$$L = n^{\frac{1}{3}} \qquad = \gamma n^{-\frac{1}{3}}$$

$\sim n^{-\frac{1}{2}}$ optimal codes

$\sim n^{-0.275}$ polar codes

If we want to design good spatially coupled codes for a given blocklength and given requirements on their error probability and decoding complexity, we need to understand how the error probability depends on the various parameters. The finite-length scaling approach which was originally introduced in the coding literature in the realm of LDPC codes by Montanari is a very useful tool in this approach. Although there is currently no rigorous proof, simulations as well as reasonable "calculations" suggest a scaling law of the form as written above, where delta is the gap to capacity and where the parameters like alpha, beta, or kappa can be determined analytically. Note that, roughly speaking, this scaling law says that spatially coupled codes scale like the underlying codes of the same blocklength as the size of each component code and that in addition we pay a moderate multiplicative penality which grows linearly in the length of the chain.

Finite-Length Scaling (BEC)

[Olmos, Urbanke, 2012]

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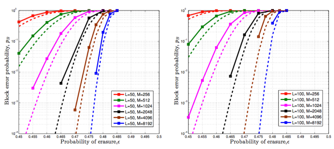

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Saturday, July 13, 13 35

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Many more...

Coupled LDPC codes: Complexity aspects of threshold saturation
M Lentmaier, GP Fettweis
Information Theory Workshop (ITW), 2011 IEEE, 668-672

Reduced complexity window decoding schedules for coupled LDPC codes
AE Pusane, M Lentmaier, GP Fettweis, DJ Costello
Information Theory Workshop (ITW), 2012 IEEE, 20-24

Connecting spatially coupled LDPC code chains
D Truhachev, DGM Mitchell, M Lentmaier, DJ Costello
Communications (ICC), 2012 IEEE International Conference on, 2176-2180

Efficient message passing scheduling for terminated LDPC convolutional codes
M Lentmaier, MM Prenda, GP Fettweis
Information Theory Proceedings (ISIT), 2011 IEEE International Symposium on ...

Comparison of LDPC block and LDPC convolutional codes based on their decoding latency
M Lentmaier, GP Fettweis
Turbo Codes and Iterative Information Processing (ISTC), 2012 7th ...

Improving spatially coupled LDPC codes by connecting chains
D Truhachev, DGM Mitchell, M Lentmaier, DJ Costello
Information Theory Proceedings (ISIT), 2012 IEEE International Symposium on ...

New codes on graphs constructed by connecting spatially coupled chains
D Truhachev, DGM Mitchell, M Lentmaier, DJ Costello
Information Theory and Applications Workshop (ITA), 2012, 392-397

Further reducing the rate-loss and complexity of decoding is an important research area currently and there are several papers on this subject. The list is in no way exhaustive. We apologize for all omissions.

Open Questions

1. Simplify, simplify, simplify, ...,
2. Spatial coupling as a proof technique (Maxwell conjecture, better bounds on K-SAT threshold, etc.)
3. Rate loss mitigation, other ways of introducing "boundary effect"? Derive fundamental lower bounds on the rate-loss.
4. Find systematic ways of designing codes (finite-length scaling, determine wave speed, ...).
5. Further applications.

Coupling and Nucleation of Crystals

One particularly insightful description why spatial coupling works was given by Krzakala, Mezard, Sausset, Sun, and Zdeborova. The threshold saturation phenomenon is equivalent to the nucleation phenomenon in physics. Nucleation explains amongst other things how crystals grow, starting with a seed or nucleus. In the video above this phenomenon is explained by looking at supercooled water. We thank Luis Salamanca for pointing out this particular YouTube video. Let us quickly explain what it shows. Assume we take a very clean container and very clean water. We can then put it into a freezer for several hours and cool it below 0 degree Celsius. If we leave it in the freezer for too long it will simply freeze, but if we keep it there only for a few hours there is a good chance that it will still be liquid despite the fact that it has a temperature below 0. The reason for this is that this supercooled water is in a **metastable** state. In this metastable state the supercooled water is not in the lowest energy state but in order to get to this state it needs a small **seed or nucleus** in order to start the crystalization process. If left alone for a long period, there is a high chance that a suitable crystal seed forms at some spot just by pure chance and if this seed is large enough the crystalization process will sweep throughout the container. But the expected time it takes for a crystal seed to form without external influence is sufficiently large that we can observe water in this supercooled form. Why does the crystal have to be large enough. In order for a small seed to grow there are two energy terms at work. First, since the crystal represents a lower form of energy, we gain by expanding an initial seed in size. This effect scales like the volume. On the other hand, we have to enlarge the boundary region between the crystal seed and the not yet crystalized water outside. This costs energy. This effect grows like the surface area. If the crystal is large enough the volume wins out and the crystal grows. But there is a critical volume below which the seed would simply collapse again.

The above phenomenon is exactly what happens for spatially coupled ensembles. Think of coding. The extra information provided at the boundary is the seed. If this is sufficiently large then the decoding wave sweeps through the structure and the decoder reaches the lower energy state, which corresponds to MAP decoding.

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The phenomenon of threshold saturation is closely connected to the way of how crystals grow.