Spatial Coupling and the Threshold Saturation Phenomenon

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EPFL

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https://dl.dropbox.com/u/2826733/ISIT%202013%20Tutorial.m4v

The latest version of these slides (Keynote and PDF) can be found at
https://ipg.epfl.ch/doku.php?id=en:publications:scc_tutorial

Part I: All we know about iterative coding we learnt by looking at the BEC

In part I and II we will talk exclusively about the binary erasure channel (BEC). This will avoid having to deal with technicalities. But we will present things in a way that all statements stay valid for the general case (general binary-input memoryless output-symmetric (BMS) channels).

Uncoupled Codes

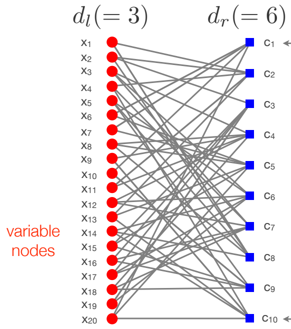
Let us start by looking at the simplest case, namely regular LDPC codes, and how to analyze them.

Introduction - Graphical Codes

Low-density Parity-Check (LDPC) Codes

Low-Density Parity-Check Codes
Robert G. Gallager
1963

$d_l(=3)$ $d_r(=6)$



$c_1 \longleftrightarrow x_6 + x_7 + x_{10} + x_{20} = 0$

$$\text{rate} \geq r_{\text{design}} = \frac{\#\text{variables} - \#\text{checks}}{\#\text{variables}} = \frac{20-10}{20} = \frac{1}{2}$$

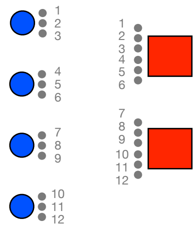
$\text{rate} \sim r_{\text{design}}$

$c_{10} \longleftrightarrow x_4 + x_9 + x_{13} + x_{14} + x_{20} = 0$

It was shown by Miller and Cohen (The rate of regular LDPC Codes, IEEE Trans. IT, 49, 2003, pp. 2989--2992), that with high probability the rate of a randomly chosen regular code is very close to this lower bound. See also page 81 in MCT (Modern Coding Theory). By *regular* code here we mean a code where all variables have degree lets say d_l and all check nodes have degree, lets say d_r.

Ensemble of Codes - Configuration Construction

(3, 6) ensemble



each configuration has uniform probability

code is sampled u.a.r. from the ensemble and used for transmission

The configuration model is a very convenient way of defining the ensemble since it is trivial to sample from this ensemble. All we need is to create a sample uniformly at random from the set of permutations on E letters, where E is the number of edges. But note that in this definition two distinct permutations can create the "same graph" once we delete the labels of the sockets. In other words, the probability distribution when we delete socket labels is no longer uniform. A further advantage of using the configuration model is that it leads to a simple analysis.

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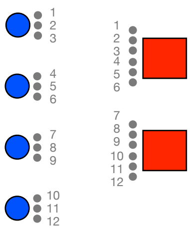

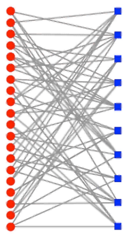

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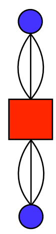
Ensemble of Codes - Protograph Construction



IPN Progress Report 42-154 August 15, 2003

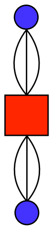
Low-Density Parity-Check (LDPC) Codes Constructed from Protographs

J. Thorpe[1]

We introduce a new class of low-density parity-check (LDPC) codes constructed from a template called a protograph. The protograph serves as a blueprint for constructing LDPC codes of arbitrary size whose performance can be predicted by analyzing the protograph. We apply standard density evolution techniques to predict the performance of large protograph codes. Finally, we use a randomized search algorithm to find good protographs.

Protographs have two advantages. First, they are a convenient and compact way of specifying graphical codes. Second, the additional structure can be useful if we want to implement codes in practice.

Ensemble of Codes - Protograph Construction



In order to create a “real” graph from a protograph we “lift” it. This means that we make M copies, where M is typically in the order of hundreds or thousands. In the example above we chose M=5. We then connect these copies by permuting the edges in each “edge bundle” by means of a permutation chosen uniformly at random.

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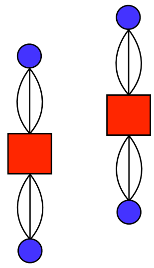

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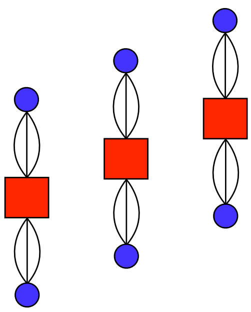

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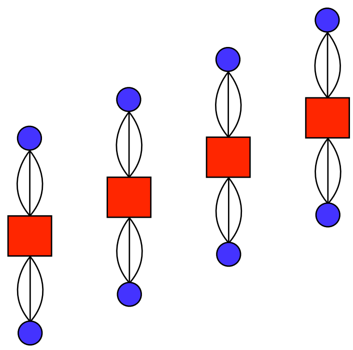

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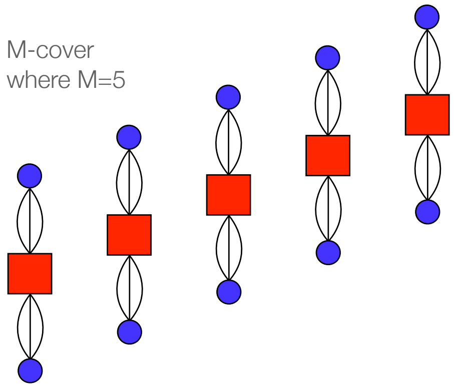

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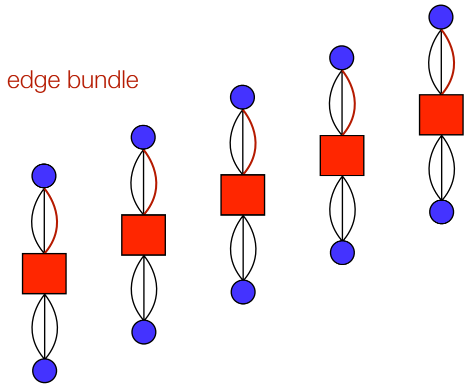

By “edge bundle” we mean here a set of “like” edges. I.e., edges which connect the same variable node and the same check node in each protograph. In the slide above a particular edge bundle is indicated in red.

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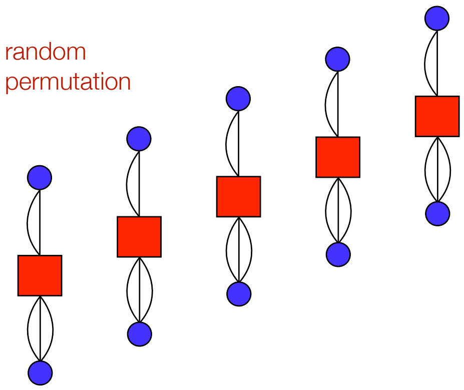

We now permute the edges in this edge bundle. We do the same thing for each edge bundle. Note that strictly speaking the ensemble created in this way is different from the ensemble created by the configuration model. But for the asymptotic analysis (density evolution -- see subsequent slides) the two models are equivalent.

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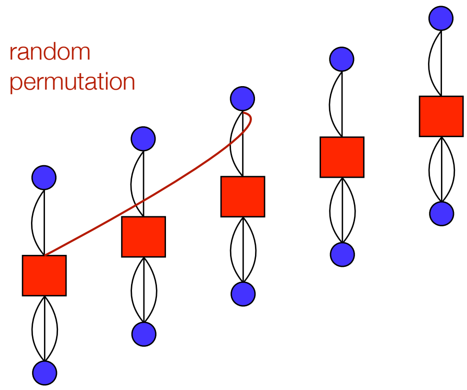

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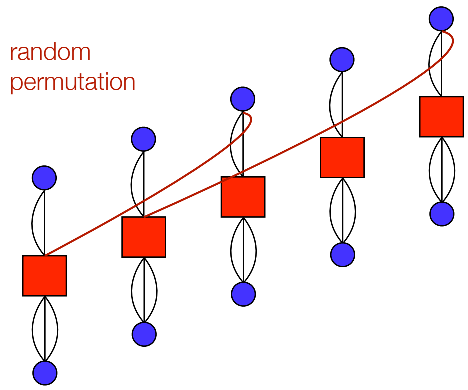

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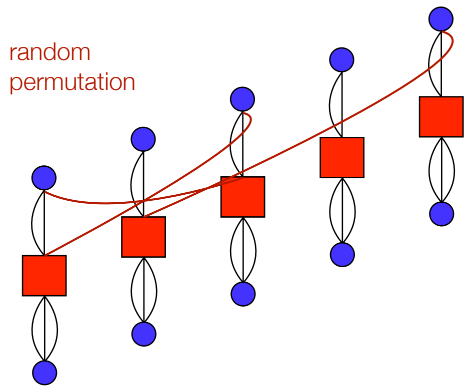

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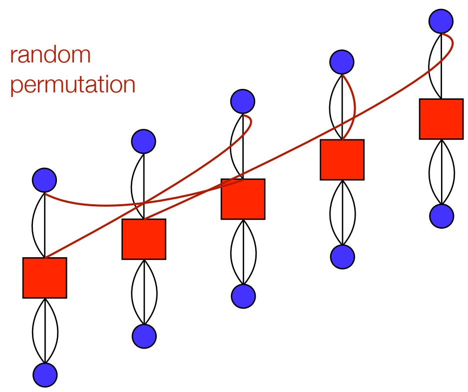

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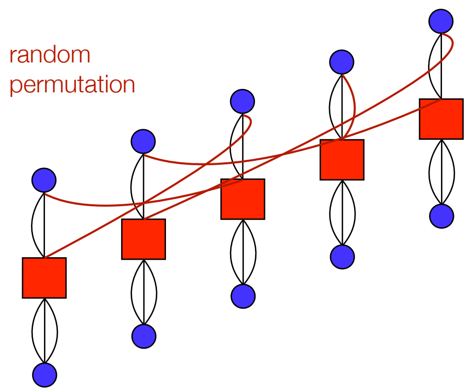

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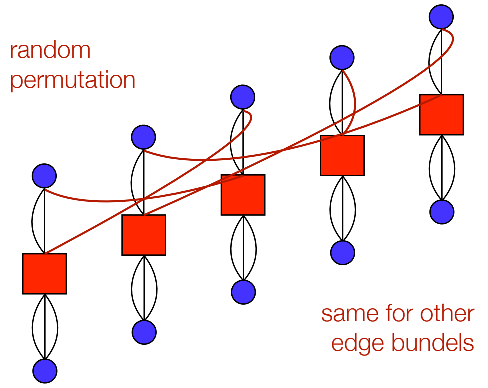

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Bit MAP Decoder to Belief Propagation Decoder

$$\begin{aligned}\hat{x}_i^{\mathrm{MAP}} &= \mathrm{argmax}_{x_i \in \{\pm 1\}} p(X_i = x_i \mid y) \\ &= \mathrm{argmax}_{x_i \in \{\pm 1\}} \sum_{x_i} \Big(\prod_j p(y_j \mid x_j) \Big) \mathbf{1}_{\{x \in \mathcal{C}\}}\end{aligned}$$

For the BEC, bit MAP decoding could be done by solving a system of linear equations, i.e., in complexity n^3.But we are interested in an algorithm that is applicable for general BMS channels (where MAP decoding is typically intractable). We therefore only consider a message-passing algorithm which is applicable also in the general case. More precisely, we consider the sum-product (also called belief-propagation (BP)) algorithm. This algorithm performs bit MAP decoding on codes on graphs whose factor graph is a tree.

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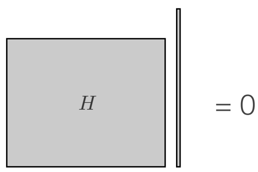

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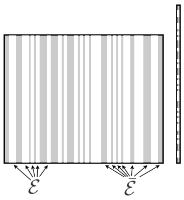

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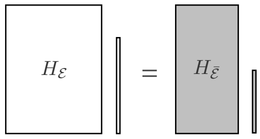

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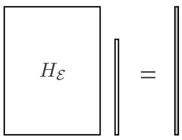

$$Hx^T = 0^T$$
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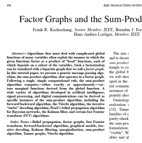

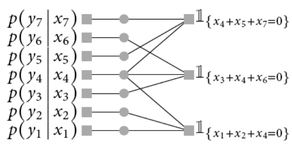

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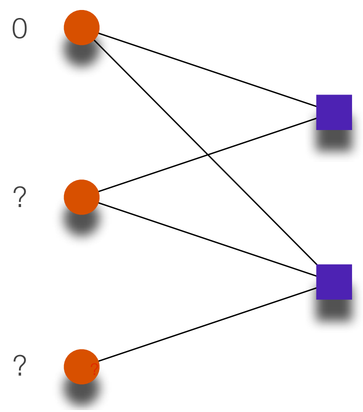

Here we see the BP algorithm in action. For the BEC the BP algorithm is particularly simple and performs a very natural operation. Every time we have a check node so that all but one of its inputs are known, the BP algorithm uses the relationship implied by this check node to determine the unknown input.

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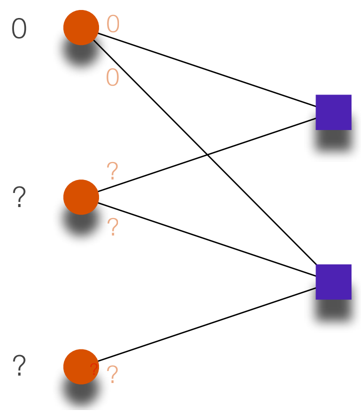

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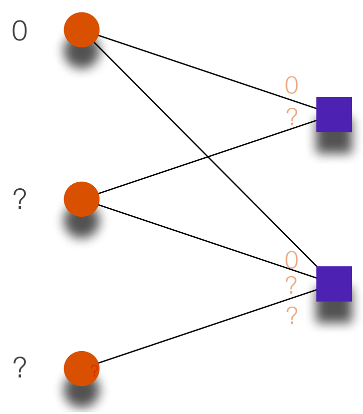

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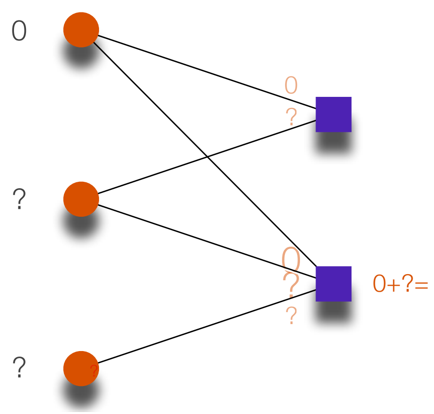

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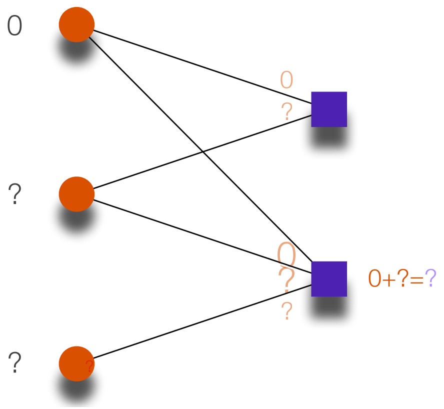

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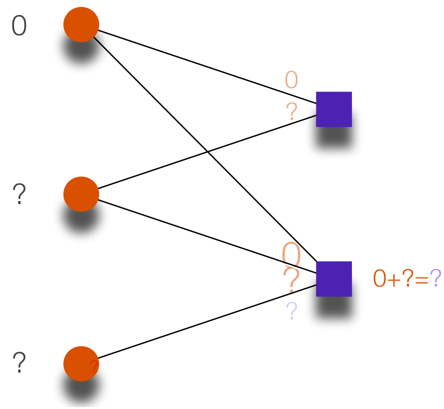

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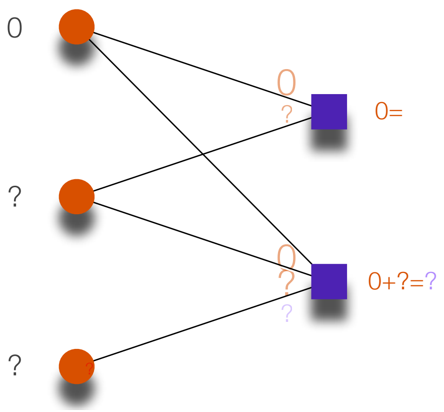

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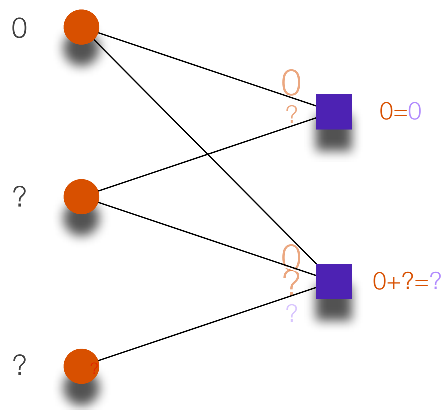

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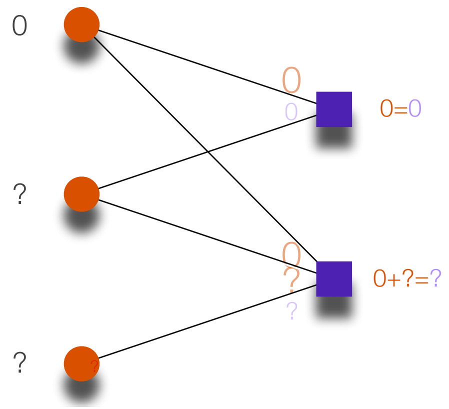

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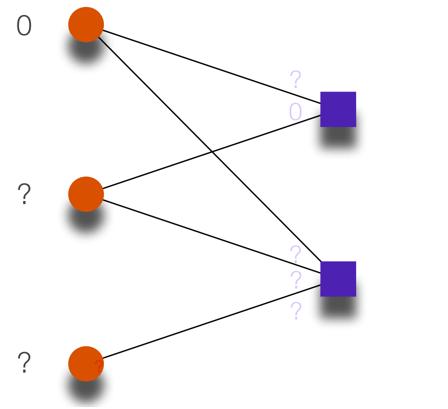

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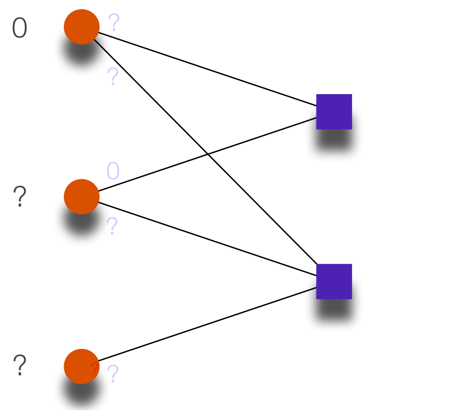

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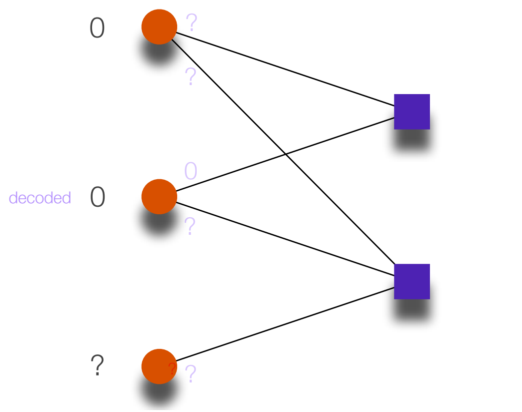

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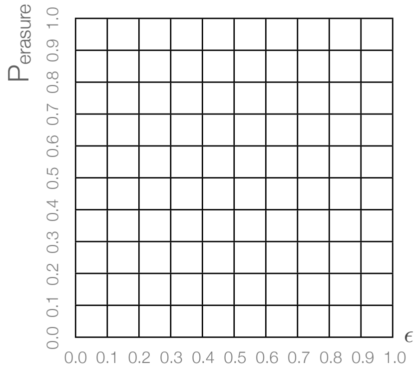
How does BP perform on the BEC?

Now where we have defined the class of codes we consider, and the algorithm which we use for decoding, we proceed to see how this combination performs.

How does BP perform on the BEC?



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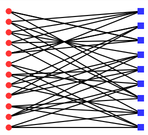

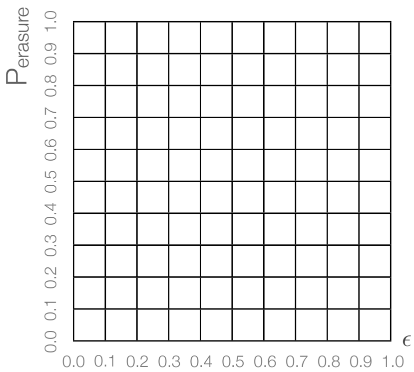

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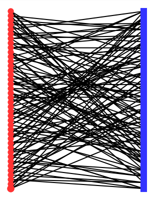

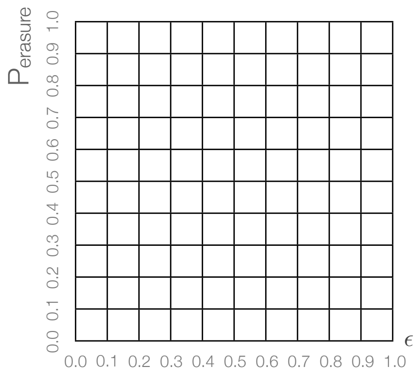

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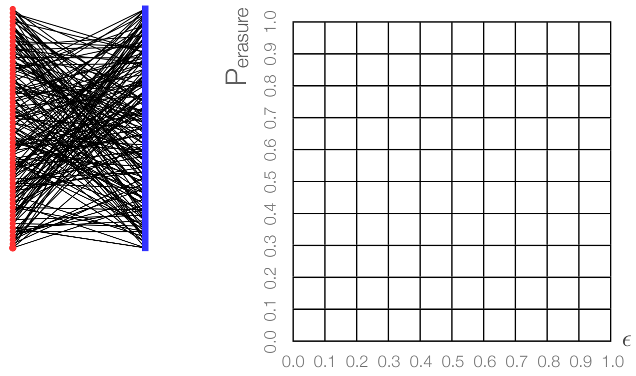

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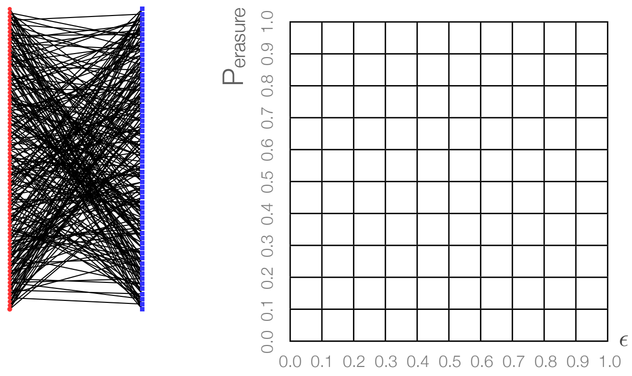

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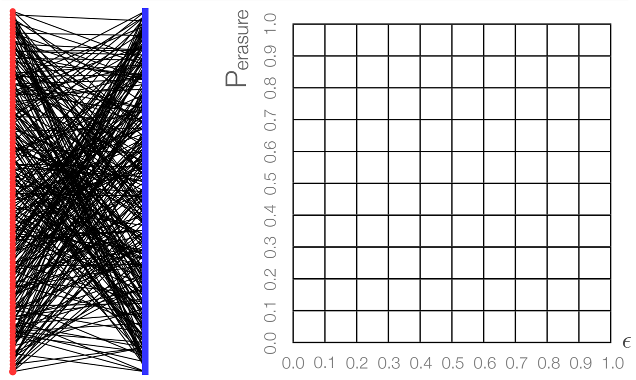

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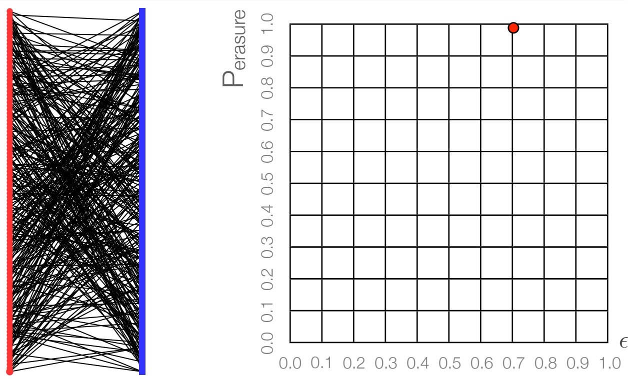

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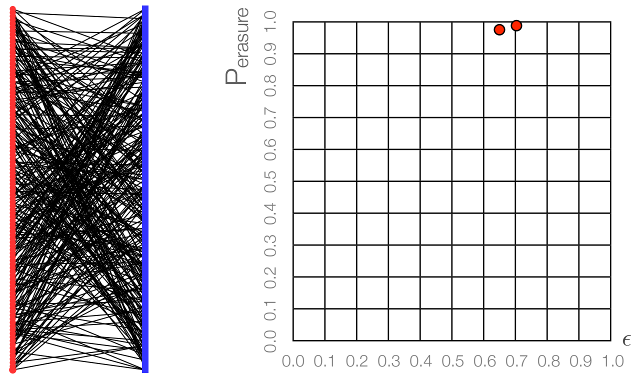

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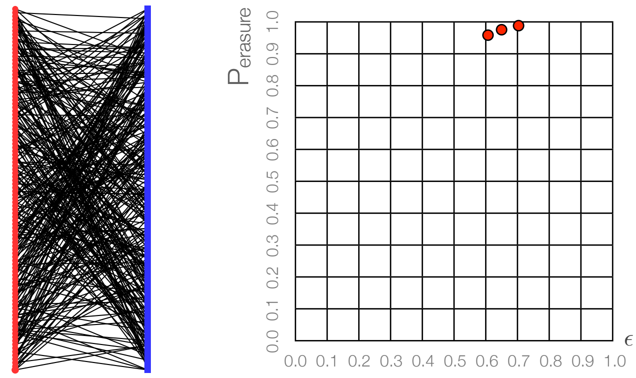

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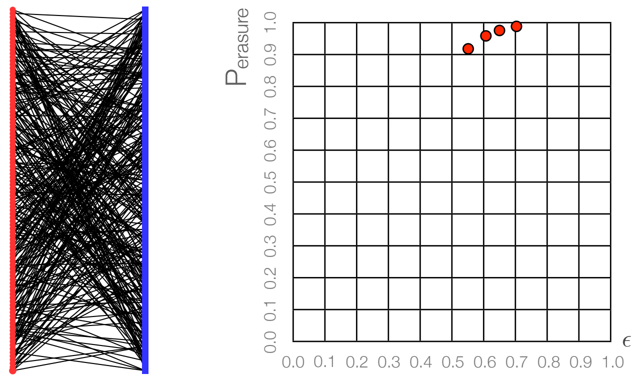

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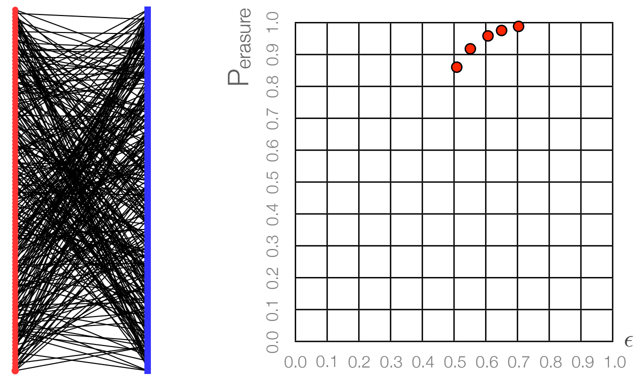

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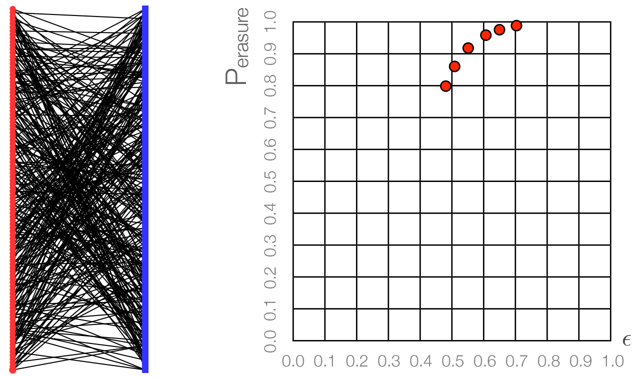

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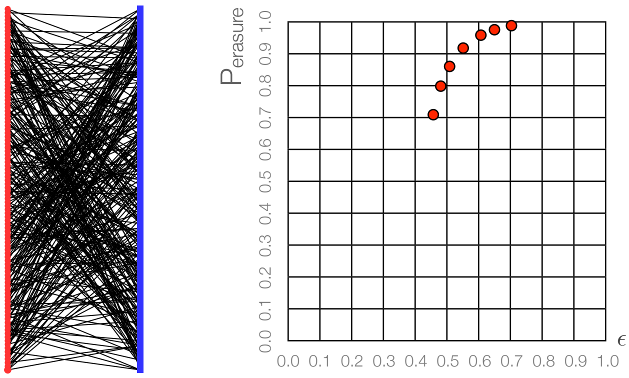

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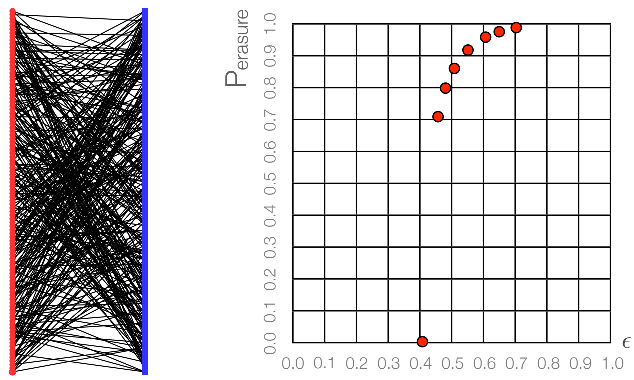

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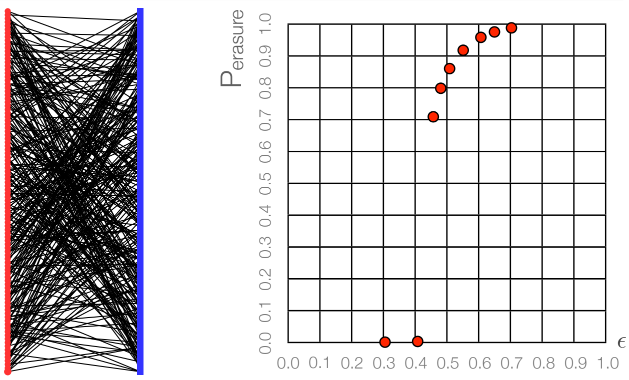

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Asymptotic Analysis - Density Evolution (DE)

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one iteration of BP at check node

Let us analyze how the erasure probability behaves during the decoding process for a large code. We do this by looking how the erasure probability behaves at each of the two types of the nodes. Consider a (d_l, d_r)-regular code, i.e., every variable node has degree d_l and and every check node has degree d_r.

At the variable node, if there is an incoming message which is not an erasure, then the variable node is exactly determined. This is because we are transmitting over the BEC and either we have perfect information or we have absolutely useless information. On the check node side, even if one incoming message is in erasure, the check node output has no way knowing whether it is 0 or 1 and hence the

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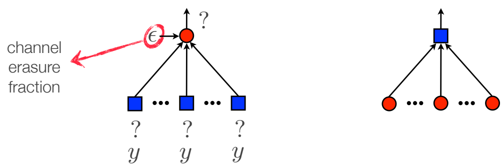

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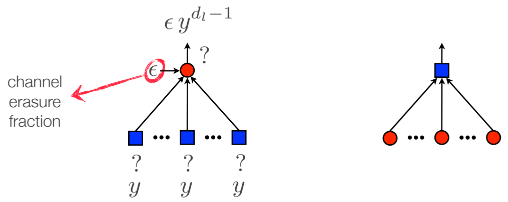

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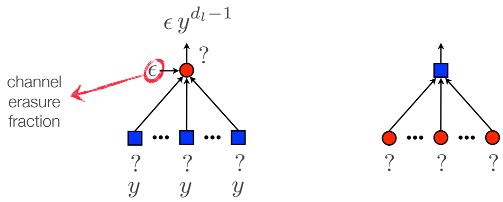

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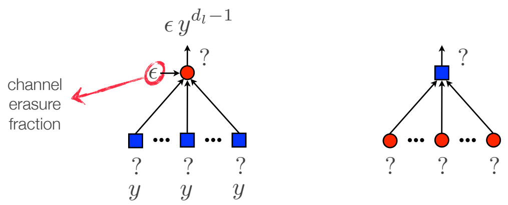

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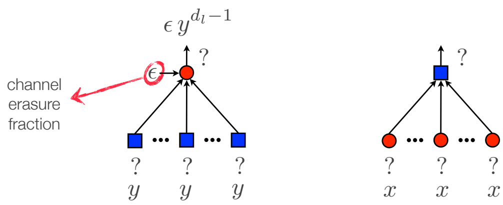

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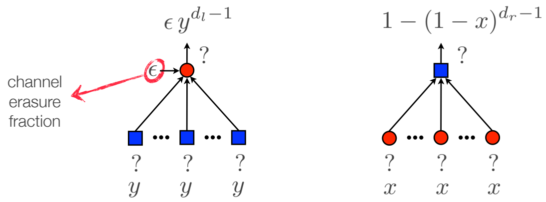

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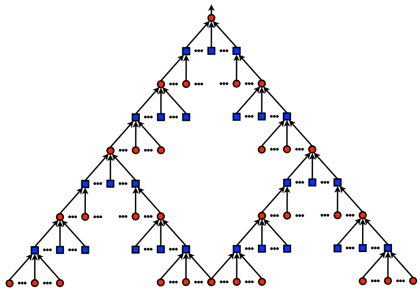

So if we perform l iterations we get a sequence of erasure probabilities. This is how Gallager analysed LDPC codes. Luby et. al. used a somewhat different procedure. In their analysis they look at the so-called peeling decoder. This decoder is entirely equivalent to the BP decoder (when transmitting over the BEC). In this decoder, as long as there is a degree-one check node, we use this check node to determine one more bit and then remove the used check node as well as the determine variable. We then follow the evolution of the graph. This can be done by writing down a system of differential equations. This method is called the Wormald method (["Differential Equations for Random Processes and Random Graphs", N. Wormald, Ann. Appl. Probability, Vol. 5, Pg. 1217-1235]).

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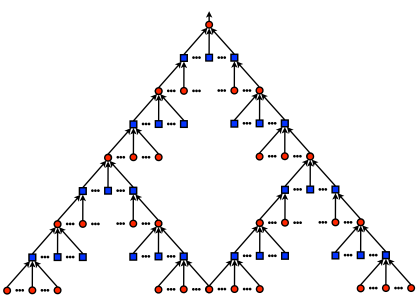

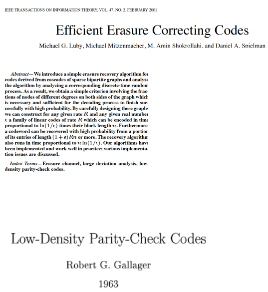

IEEE TRANSACTIONS ON INFORMATION THEORY, VOL. 47, NO. 2, FEBRUARY 2001

Efficient Erasure Correcting Codes

Michael G. Luby, Michael Mitzenmacher, M. Amin Shokrollahi, and Daniel A. Spielman

Abstract—We introduce a simple erasure recovery algorithm fo codes derived from cascades of sparse bipartite graphs and analyz the algorithm by analyzing a corresponding discrete-time randon process. As a result, we obtain a simple criterion involving the frac tions of nodes of different degrees on both sides of the graph whicl is necessary and sufficient for the decoding process to finish suc cessfully with high probability. By carefully designing these graph we can construct for any given rate R and any given real numbe ϵ a family of linear codes of rate R which can be encoded in tim proportional to $\ln(1/\epsilon)$ times their block length n. Furthermore a codeword can be recovered with high probability from a portion of its entries of length $(1+\epsilon)Rn$ or more. The recovery algorithn also runs in time proportional to $n\ln(1/\epsilon)$. Our algorithms hav been implemented and work well in practice; various implementa tion issues are discussed.

Index Terms—Erasure channel, large deviation analysis, low density parity-check codes.

Low-Density Parity-Check Codes

Robert G. Gallager

1963

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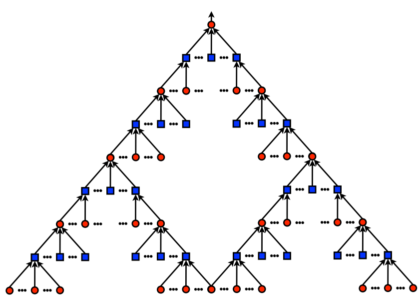
Asymptotic Analysis - Density Evolution (DE)



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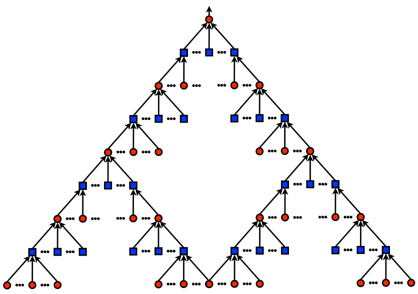

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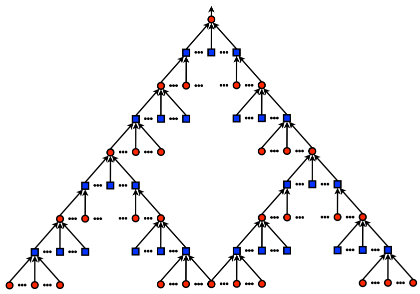

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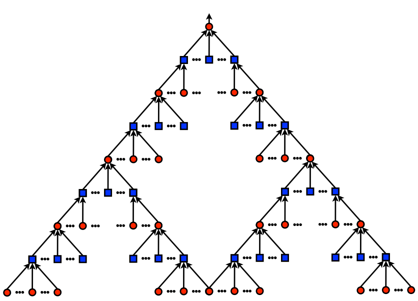

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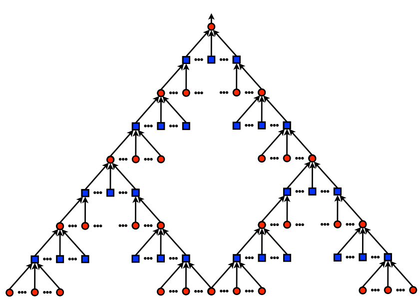

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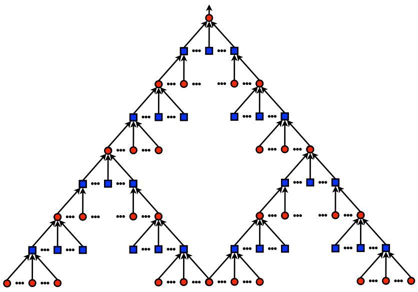

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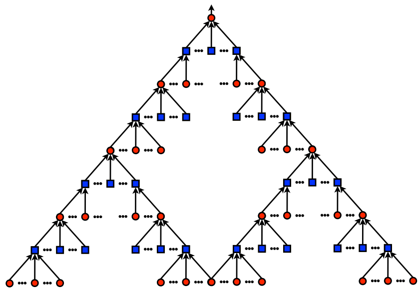

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Asymptotic Analysis - Density Evolution (DE)

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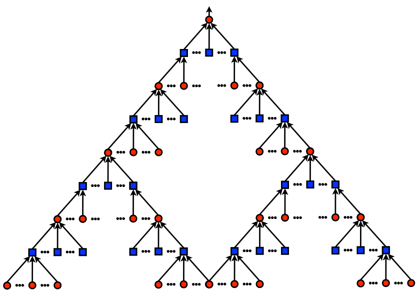

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Asymptotic Analysis - Density Evolution (DE)

Note: DE sequence is decreasing and bounded from below ⇒ converges

Note that in the density evolution approach we assume that we first fix the number of iterations and let the length of the code tend to infinity (so that there are no loops in the graph up to the desired size). We THEN let the number of iterations tend to infinity. In the Wormald approach on the other hand we take exactly the opposite limit. Luckily both approaches give exactly the same threshold. It is then easy to see that in fact we can take the limit in any order, or jointly, and that we always get the same threshold.

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Concentration: It can be shown that the behavior of almost all codes in the ensemble is close to the average predicted by DE

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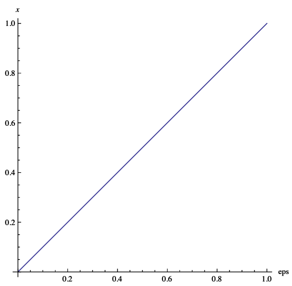
DE for (3, 6) Ensemble

erasure fraction as a function of increasing iterations for a given channel value



Let us now apply DE for our running example. We see that up to the “BP threshold”, which for the running example is around 0.429, the erasure probability tends to zero if we let the number of iterations tend to infinity. For higher values of eps the x-value tends to a non-zero value.

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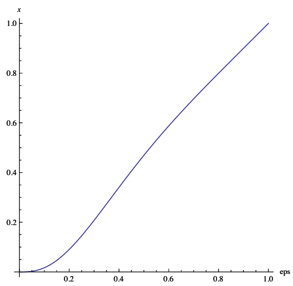

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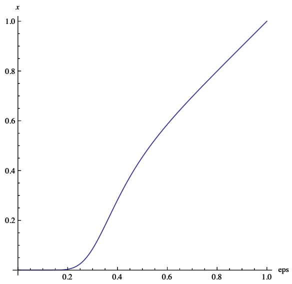

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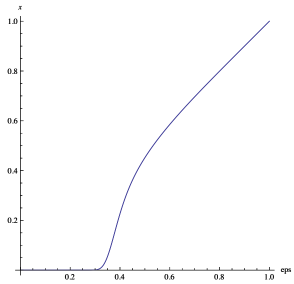

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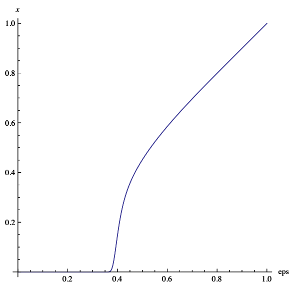

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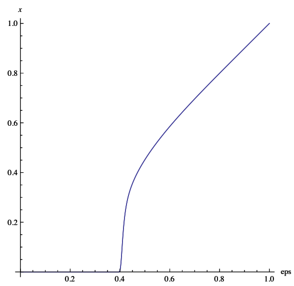

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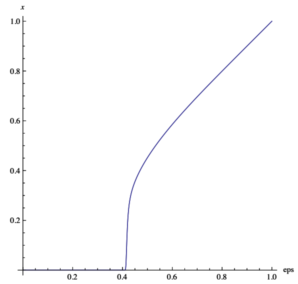

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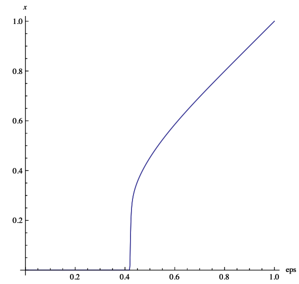

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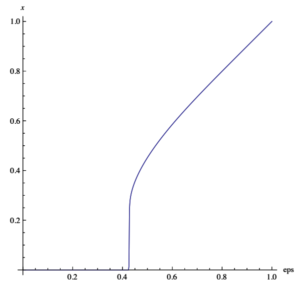

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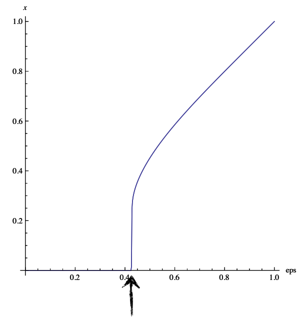

BP successful below $\epsilon \approx 0.429$

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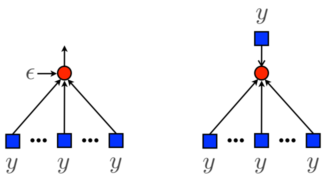
x versus EXIT

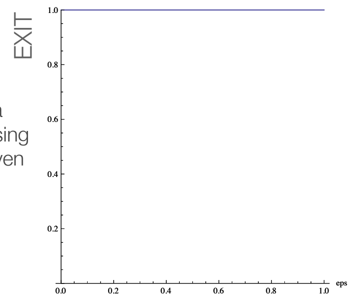
$$x = \epsilon y^{d_l - 1} \qquad \mathrm{EXIT} = y^{d_l}$$



Instead of plotting the “x-value” on the vertical axis it is often more convenient to plot the EXIT value. The EXIT value has a simple interpretation. It is the error probability of the best estimate we can do using all the “internal” messages at a node but without the channel observation at this bit. This is why we have y to the power dl and not dl-1 but we do not have the factor eps corresponding to the channel erasure fraction.

EXIT Curve for (3, 6) Ensemble

EXIT value as a function of increasing iterations for a given channel value



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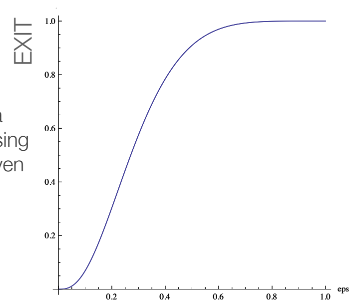

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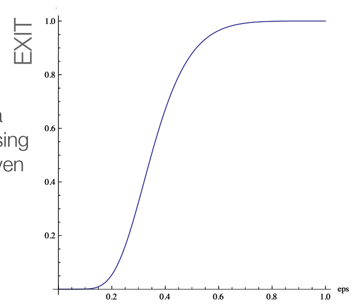

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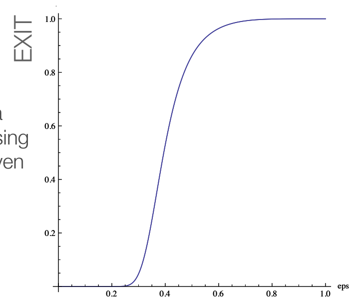

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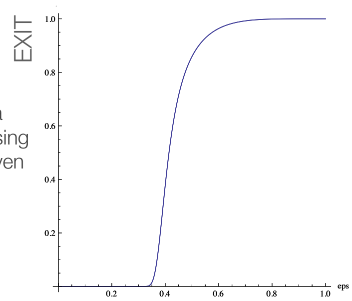

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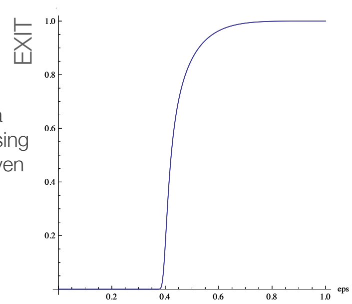

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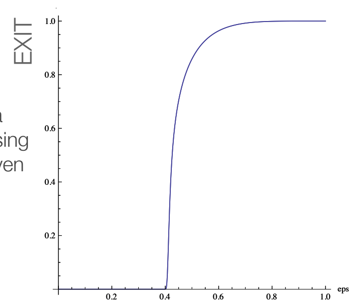

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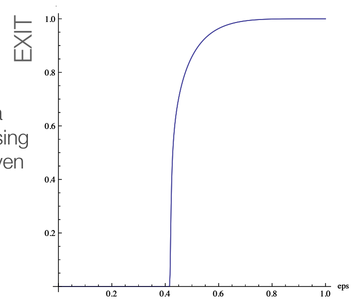

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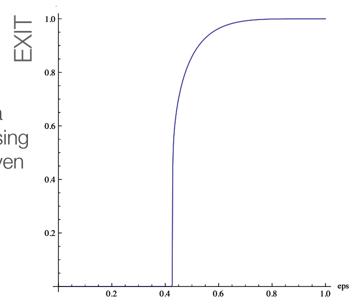

Saturday, July 13, 13 20

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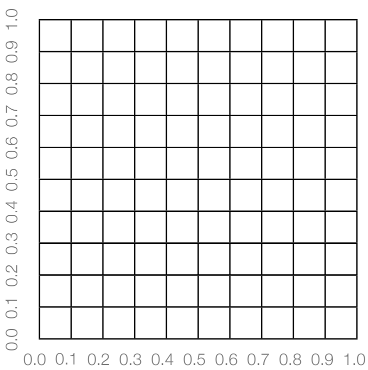
A look back ...



(3, 6) ensemble

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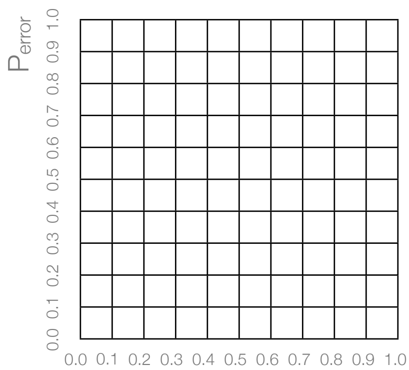

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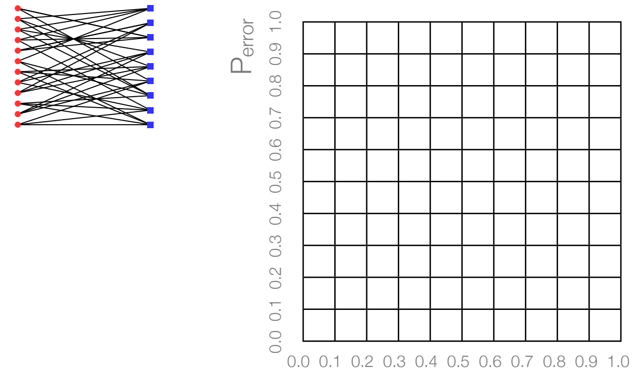

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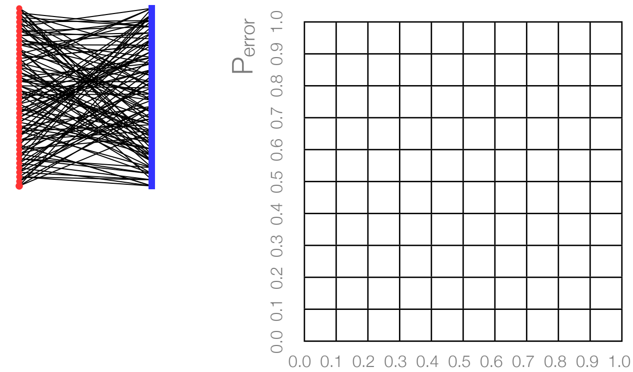

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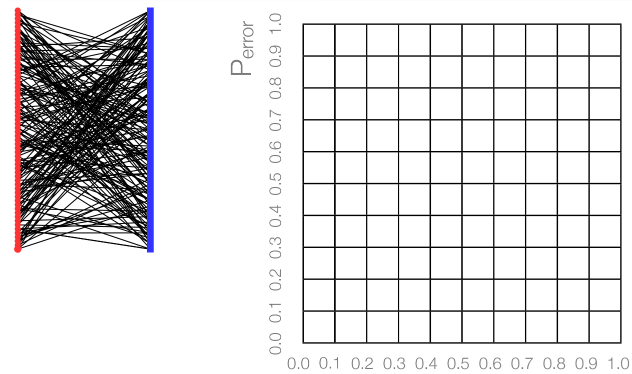

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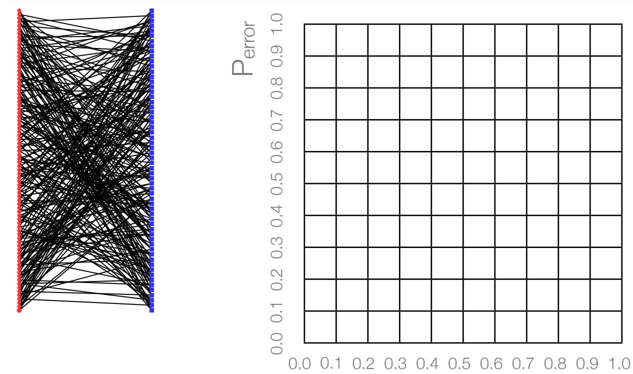

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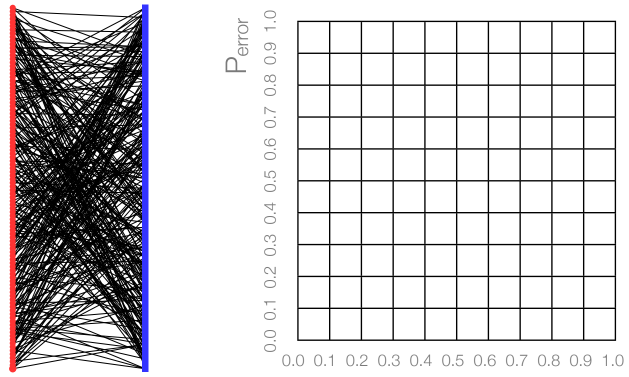

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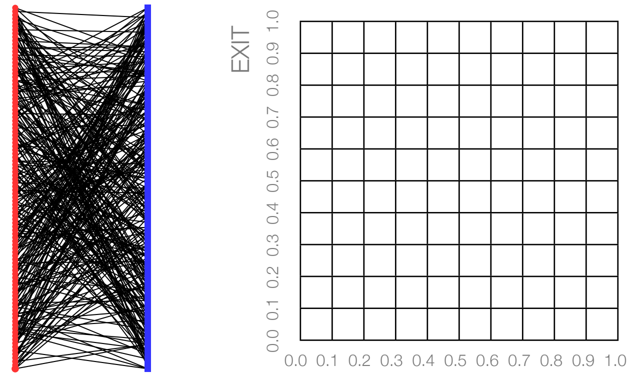

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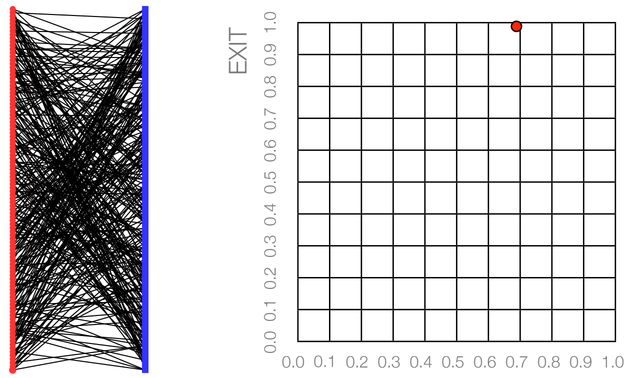

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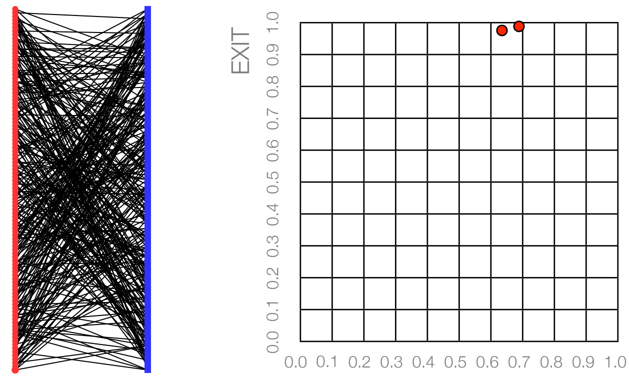

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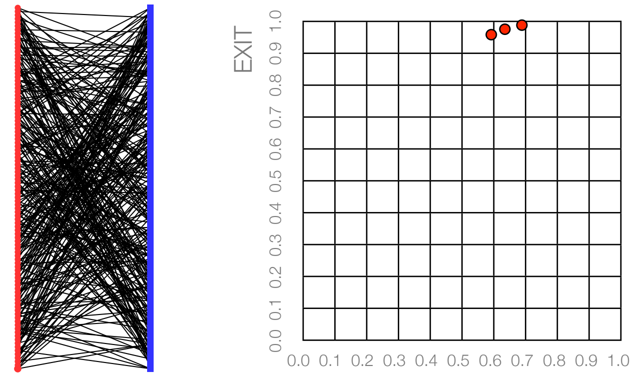

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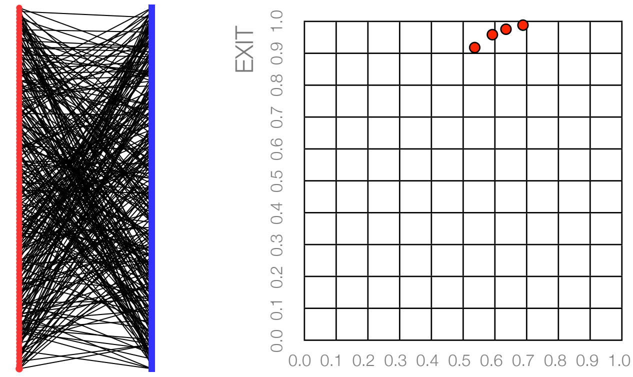

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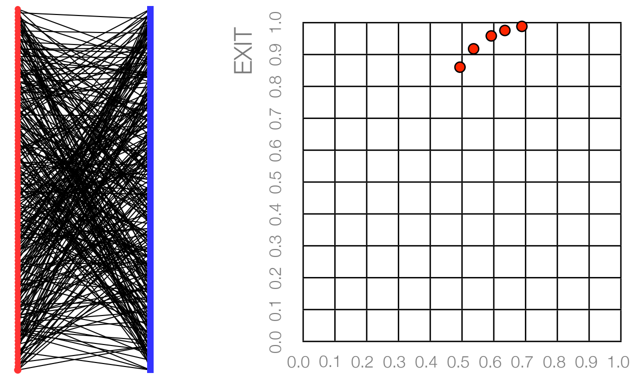

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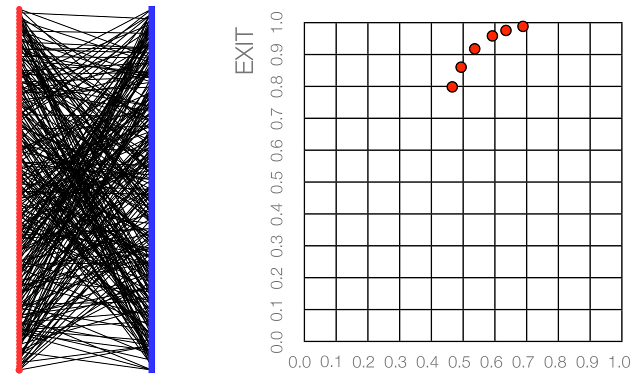

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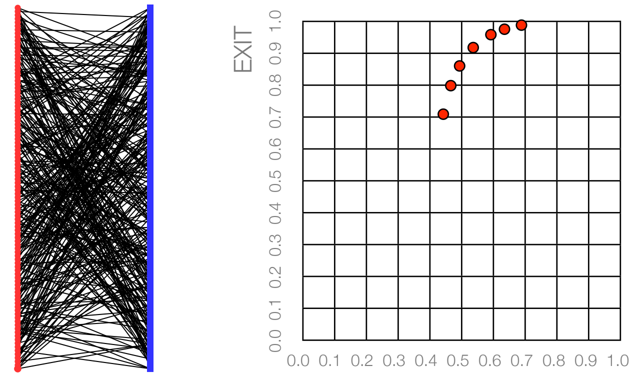

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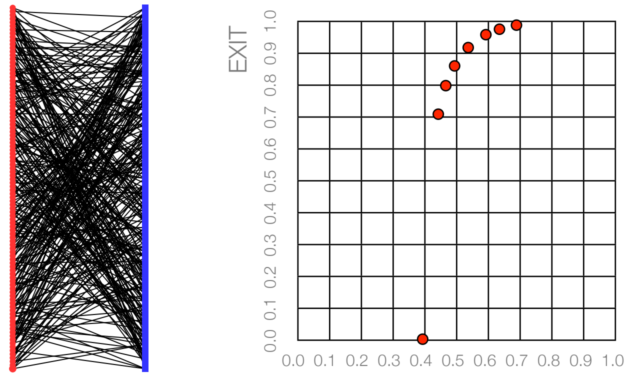

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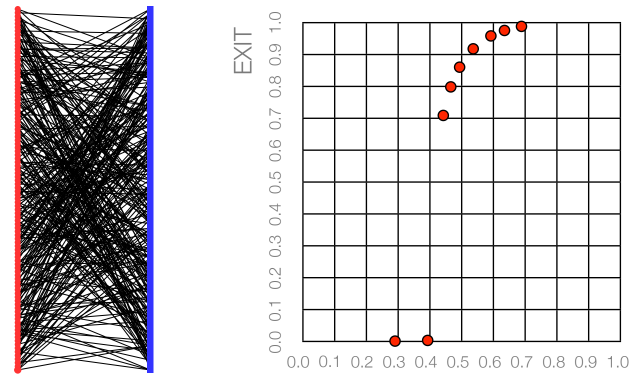

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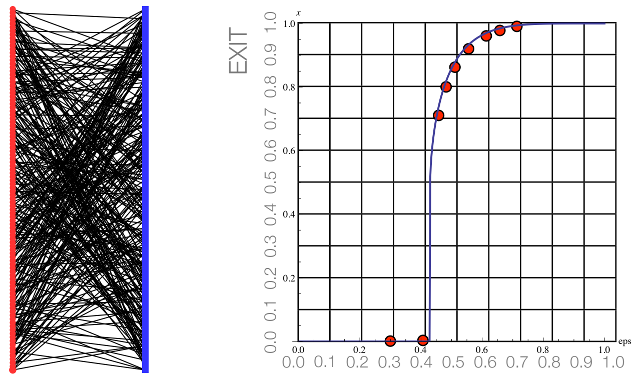

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Static Analysis via Fixed points

Forward Fixed points of DE

$$x^{(\ell=0)} = \epsilon$$

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$$x^{(\ell)} = \epsilon (y^{(\ell)})^{d_l - 1}$$

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Saturday, July 13, 13 22

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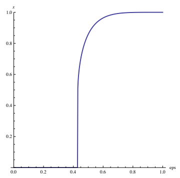

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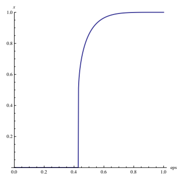

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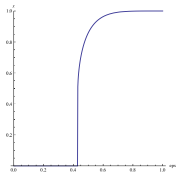

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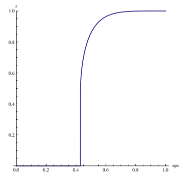

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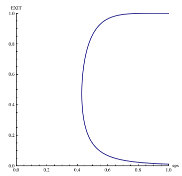

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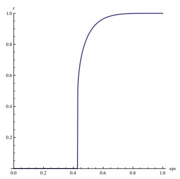

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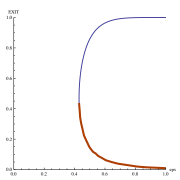

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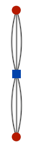
Coupled Codes

We now get to the topic of this tutorial, namely spatially coupled codes. We start with their construction and then proceed towards their analysis. We will see how these codes are intimately tied to the standard LDPC codes we have introduced till now.

Coupled Ensembles - Protograph Construction



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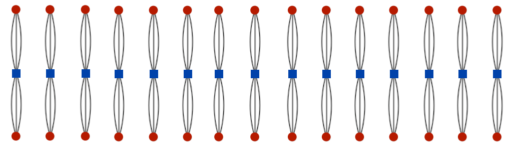

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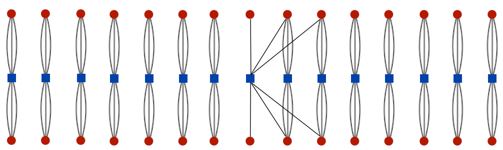

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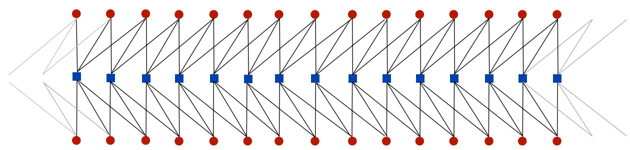

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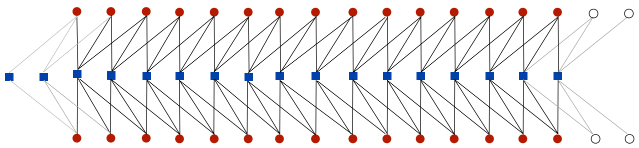

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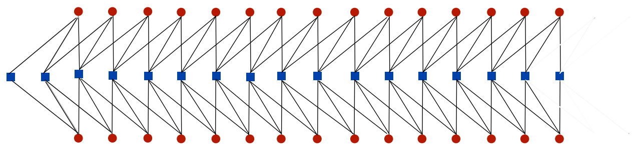

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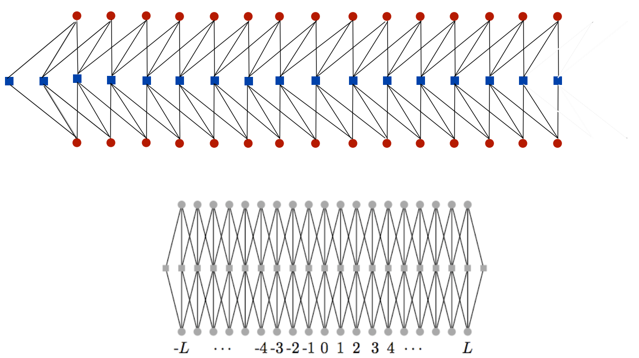

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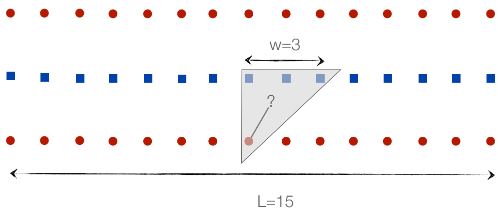
Coupled Ensembles - Random Construction



In the random version, for each edge we pick a position within a window of size w. The only restriction is that the global edge counts have to match up. But for large instance this only imposes a negligible global restriction. It is not hard to see that if we allow the window to span the entire spatial length, we will recover the standard uncoupled LDPC code.

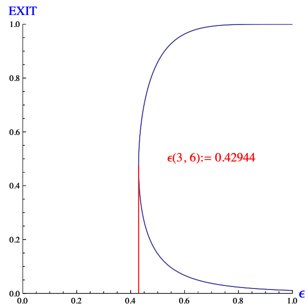
Coupled Ensembles

Protograph construction: good performance
suitable for implementation

Random construction: good for proofs

If we implement such codes in “practice”, protographs are the better choice. They in fact behave better, due to their decreased randomness. Further, the additional structure makes them well-suited for implementations. The random code construction on the other hand is better suited for proofs.

Why coupling *might* help



(d_l, d_r =2d_l)-regular
uncoupled ensemble
d_l increasing; BEC

Before we get to a more serious analysis let us see why spatial coupling might help. Let us first consider what happens if we take uncoupled ensembles and if we keep the rate fixed but increase the degrees. As we see in the sequence of EXIT curves (and as one can easily prove) the threshold decreases to zero as we increase the degrees.

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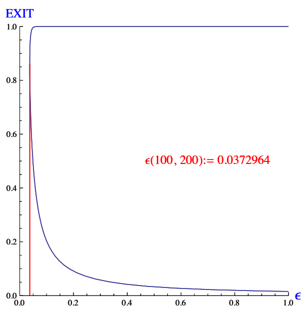

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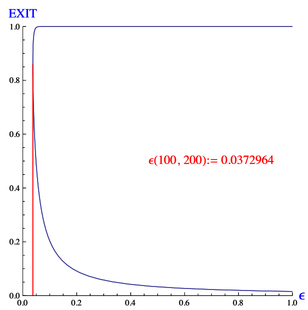

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General:

$$\mathtt{h}^{\text{BP}}(d_l, d_r, \{c_h\}) \le \frac{h_2\left(\frac{1}{2\sqrt{d_r-1}}\right)}{1-((1-r)d_r)e^{-2\sqrt{d_r-1}}}.$$

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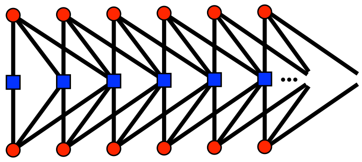

Now let us look what happens if we do the same thing for coupled ensemble. We look here at coupled ensembles constructed via the protograph approach. Further, to make the argument simple, we assume that all the edges are “double” edges, i.e., for every indicated edge above there are in fact two edges.

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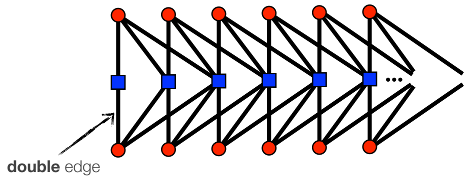

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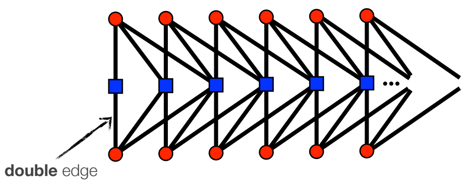
(6, 12)-regular coupled ensemble



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Why coupling *might* help

When we decode we can always ignore some additional information. This only makes the decoder perform worse. But as we see in the above case, at the boundary the code contains a (2, 4) cycle code. This code is know to have a BP threshold of 1/4 when transmitting over the BEC. So we can decode just the boundary nodes if we transmit over a BEC with erasure probability at most 1/4 using the BP decoder.

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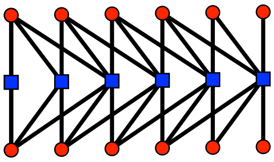

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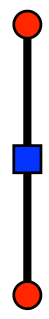

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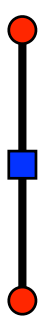

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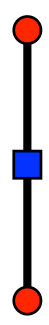

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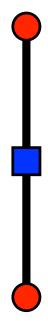

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$(2, 4)$ cycle code with threshold $\epsilon^{\mathrm{BP}} = \frac{1}{3}$

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then the first section will decode correctly whp on its own

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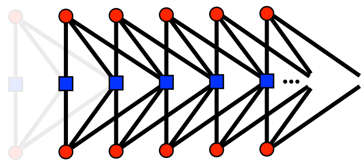
Why coupling *might* help

(6, 12)-regular coupled ensemble



looks just like before; we conclude that the threshold is at at least 1/3

But now where we decoded the boundary nodes we see that the remainder of the code looks just like the original code, except that we “chopped” off the left-most section. The code is “self-similar” in this sense. So we can continue in recursive fashion and now decode the second section and so on. We conclude that this ensemble has a BP threshold of at least 1/3.

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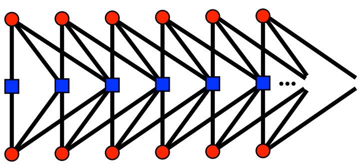

But the same argument holds if we do not start with a (6, 12) ensemble but with any (2 k, 4 k) ensemble, regardless how large the degrees are. So the BP threshold does NOT tend to zero for coupled ensembles if we increase the degrees. Indeed, we will see that they in fact get BETTER.

Why coupling *might* help

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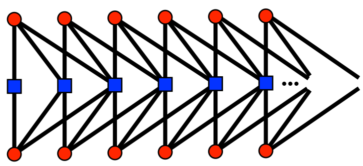
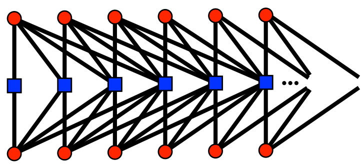


(8, 16)-regular coupled ensemble

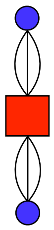


but the same argument holds even if we increase degrees

therefore threshold is lower bounded by 1/3 even if the degrees tend to infinity ....

But the same argument holds if we do not start with a (6, 12) ensemble but with any (2 k, 4 k) ensemble, regardless how large the degrees are. So the BP threshold does NOT tend to zero for coupled ensembles if we increase the degrees. Indeed, we will see that they in fact get BETTER.

Recall DE for the uncoupled case

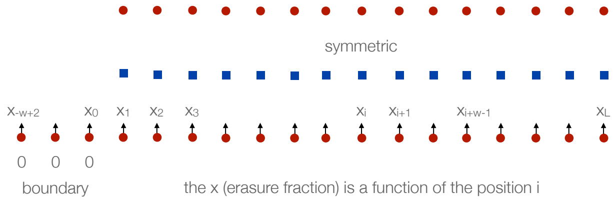


$$x^{(\ell)} = \epsilon(y^{(\ell)})^{d_l - 1}$$

$$y^{(\ell)} = 1 - (1 - x^{(\ell-1)})^{d_r - 1}$$

Let us write down now the DE equations for the coupled case. First recall that for the uncoupled case the "state" used in DE is a single scalar, namely x, which represents the erasure fraction along an outgoing edge from the variable node.

DE for the Coupled Case - Random Construction



For the coupled case the state is no longer a scalar. In the "interior" of the chain the structure of the code is shift invariant (i.e., looks the same for each position) but at the boundary the conditions are no longer uniform. We hence need one scalar x_i to describe the state at each position i. Why is the boundary set to 0. We assume that at the boundary we *know* the values. So hence the erasure probability at the boundary is 0.

DE for the Coupled Case - Random Construction

$$x_i = \epsilon(\frac{1}{w}\sum_{j=0}^{w-1} y_{i+j})^{d_l-1} \qquad y_i = 1 - (1 - \frac{1}{w}\sum_{k=0}^{w-1} x_{i-k})^{d_r-1}$$

$$y_i, \cdots, y_{i+w-1} \qquad x_i, \cdots, x_{i-w+1}$$

Again we look at what happens at the two nodes. The equations are very similar to the uncoupled case but there is one difference. If we look e.g.at the random case then each edge can be connected to positions in a certain range. We therefore need to average over the incoming messages from this range. Once we have done the average, we proceed as in the uncoupled case. Note that with respect to the way we defined the random ensemble, variables are always connected to position "to the right" and check nodes are always connected to variable nodes "on the left."

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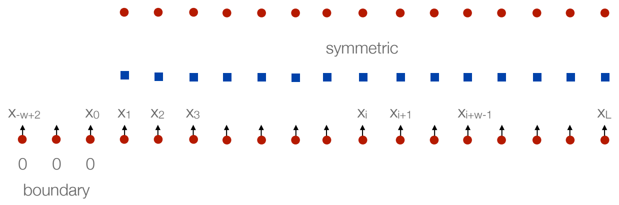
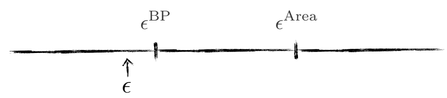


$$x_i^{(\ell)} = \epsilon\Big(1 - \frac{1}{w}\sum_{j=0}^{w-1}(1 - \frac{1}{w}\sum_{k=0}^{w-1} x_{i+j-k}^{(\ell-1)})^{d_r-1}\Big)^{d_l-1}$$

If we put the two parts together we get now the recursive DE equations for the coupled chain. Note that we now have as many x_i values as the length of the chain and that the various equations for the x_i values are "coupled" through the averaging operations.

DE for Coupled Ensemble



Let us now see how coupled codes perform. Let us first run DE for an eps value below the BP threshold of the uncoupled case. Note that except at the boundary, coupled codes have exactly the same local connectivity as the uncoupled ensemble they are based on. At the boundary they are "better" due to the known variables there. So we expect them to behave no worse than their uncoupled sisters. In the movie above we plot the x_i values as a function of the iteration. For the case above, the chain has length 100. Indeed, as we see from this movie, DE proceeds exactly as for the uncoupled case if we look at the x_i values in the center of the chain. At the boundary we see somewhat better values due to the termination. And as expected, the DE is able to drive the erasure fraction in each section to zero and BP is successful.

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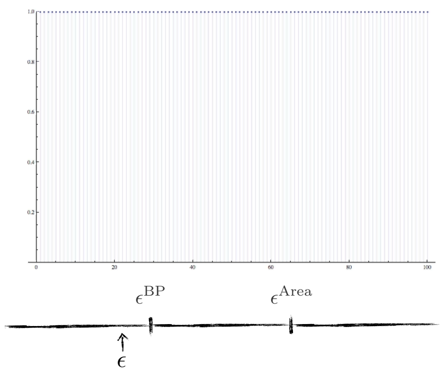

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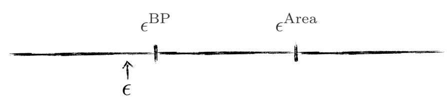

Now let us do the same experiment for a value above the BP threshold of the uncoupled ensemble but not too much larger. As we see we get a very interesting behavior. After a few iterations where all the "obvious" deductions are being made the decoder still proceeds to make progress. On both ends a small "wave front" has formed. These wave fronts move towards the center at a constant speed. Note that the wave connects the two FPs which exist also for the uncoupled case. Namely, the undesired FP in which the uncoupled code gets stuck (forward FP of DE) and the desired FP (namely zero) which an optimal decoder would find. The wave "bridges" these two FPs and thus guides nodes in the interior of the chain towards the desired FP. Also, note the special structure that the erasure fraction at each section forms:

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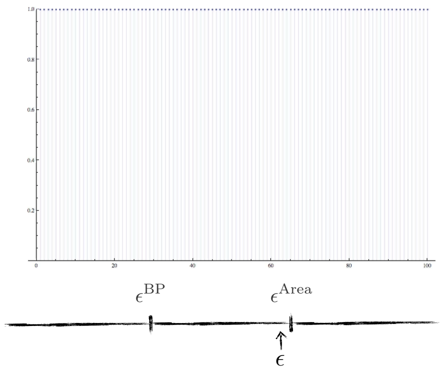

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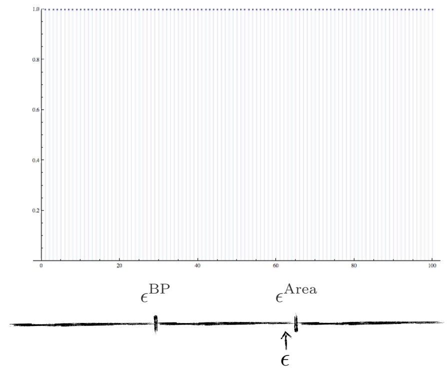

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Finally, if we go above a certain value of eps and we run DE then we get a non-trivial FP. Note that in the middle of the chain the xi values are exactly as large as they would be for the same eps value in the uncoupled case. Only at the boundary do we get somewhat better values due to the termination.

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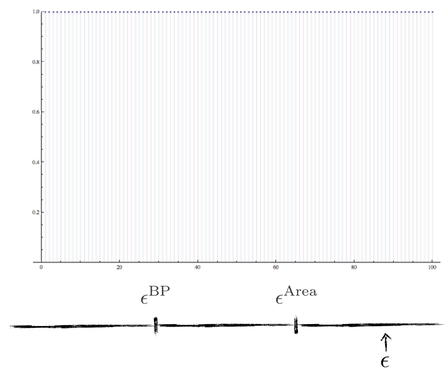

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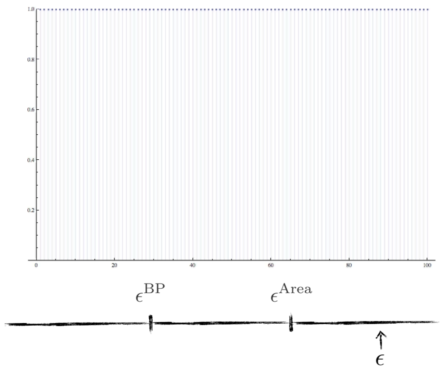

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Questions

What is ϵ^{Area} ?

Why does this happen?

Spoiler alert: We will see that the area threshold is essentially equal to the MAP threshold of the underlying ensemble! Here, the MAP threshold is the threshold an optimal decoder would achieve. And it turns out to be the BP threshold of the coupled code ensemble!

Given what we have seen, we are faced with a few questions. First, what is this parameter $\varepsilon^{\mathrm{Area}}$ up to which spatially coupled ensembles can decode? Second, why does the system so drastically change its behavior when we couple it? Spoiler Alert: We will see that the answer to the first question is that $\varepsilon^{\mathrm{Area}}$ is essentially equal $\varepsilon^{\mathrm{MAP}}$ of the underlying ensemble! We say here essentially since, as we will explain in more detail later, this is strictly true only when we let certain quantities tend to infinity. But even if we do not let these quantities tend to infinity but choose them reasonably small, the difference is typically very very small.

Historical Outline of Spatially Coupled Codes

Big bang:

A. J. Felström and K. S. Zigangirov, "Time-varying periodic convolutional codes with low-density parity-check matrix," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 2181–2190, Sept. 1999.

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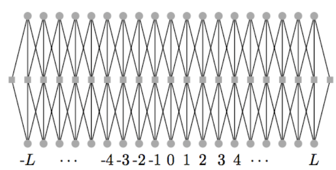

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Variations, code word and pseudo code word analysis:

K. Engdahl and K. S. Zigangirov, “On the theory of low density convolutional codes I,” *Problemy Peredachi Informatsii*, vol. 35, no. 4, pp. 295–310, 1999.

M. Lentmaier, D. V. Truhachev, and K. S. Zigangirov, “To the theory of low-density convolutional codes. II,” *Probl. Inf. Transm.*, vol. 37, no. 4, pp. 288–306, 2001.

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——, “Pseudocodeword performance analysis for LDPC convolutional codes,” *IEEE Trans. Inform. Theory*, vol. 55, no. 6, pp. 2577–2598, June 2009.

K. Engdahl, M. Lentmaier, and K. S. Zigangirov, “On the theory of low-density convolutional codes,” in *AAECC-13: Proceedings of the 13th International Symposium on Applied Algebra, Algebraic Algorithms and Error-Correcting Codes*. London, UK: Springer-Verlag, 1999, pp. 77–86.

R. M. Tanner, D. Sridhara, A. Sridharan, T. E. Fuja, and D. J. Costello, Jr., “LDPC block and convolutional codes based on circulant matrices,” *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 2966 – 2984, Dec. 2004.

D. G. M. Mitchell, A. E. Pusane, K. S. Zigangirov, and D. J. Costello, Jr., “Asymptotically good LDPC convolutional codes based on protographs,” in *Proc. of the IEEE Int. Symposium on Inform. Theory*, Toronto, CA, July 2008, pp. 1030 – 1034.

——, “Pseudocodeword performance analysis for LDPC convolutional codes,” *IEEE Trans. Inform. Theory*, vol. 55, no. 6, pp. 2577–2598, June 2009.

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Termination and density evolution analysis:

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M. Lentmaier, A. Sridharan, K. S. Zigangirov, and D. J. Costello, Jr., “Terminated LDPC convolutional codes with thresholds close to capacity,” *CoRR*, vol. abs/cs/0508030, 2005.

M. Lentmaier and G. P. Fettweis, “On the thresholds of generalized LDPC convolutional codes based on protographs,” in *Proc. of the IEEE Int. Symposium on Inform. Theory*, Austin, USA, 2010.

M. Lentmaier, A. Sridharan, K. S. Zigangirov, and D. J. Costello, Jr., “Iterative decoding threshold analysis for LDPC convolutional codes,” *IEEE Trans. Info. Theory*, Oct. 2010. Vol. 56, No. 10, pp. 5274

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M. Lentmaier, A. Sridharan, K. S. Zigangirov, and D. J. Costello, Jr., “Terminated LDPC convolutional codes with thresholds close to capacity,” *CoRR*, vol. abs/cs/0508030, 2005.

M. Lentmaier and G. P. Fettweis, “On the thresholds of generalized LDPC convolutional codes based on protographs,” in *Proc. of the IEEE Int. Symposium on Inform. Theory*, Austin, USA, 2010.

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Historical Outline of Spatially Coupled Codes

Maxwell Construction, MAP Thresholds, generalized EXIT functions

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Historical Outline of Spatially Coupled Codes

Threshold saturation effect and proof for the BEC:

S. Kudekar, T. Richardson, and R. Urbanke, “Threshold Saturation via Spatial Coupling: Why Convolutional LDPC Ensembles Perform so well over the BEC,” *IEEE Trans. Inform. Theory*, vol. 57, no. 2, pp. 803–834, Feb. 2011.

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Main Message

1. In order to get exactly ϵ^{MAP} we have to let the “connection width” w tend to infinity. But in practice even small widths like w=3 lead to thresholds which are almost indistinguishable ϵ^{MAP}. (e.g., for the (3, 6) case and the BEC the difference is about 10^{-5}.
2. The MAP threshold is an *increasing* function of the degrees and converges to the Shannon threshold exponentially fast. This is contrary to the BP threshold for uncoupled codes which typically *decreases* in the degree.
3. We will see that using spatial coupling we can construct codes which are capacity-achieving *universally* across the whole set of BMS channels.
4. On the downside, due to the termination which is required, we loose in rate. We hence have to take the chain length large enough in order to amortize this rate loss. Therefore, the blocklength has to be reasonably large.

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(not exhaustive) References

Most papers on this topic can be found on arXiv.org. To get started, look for "spatial" and "coupl" in the title and restrict your search to the time range since 2009 and the Computer Science category. Currently you will find 45 articles on this topic on arXiv.org. But there are many more ... e.g., many important papers which use spatial coupling in the area of compressive sensing do not contain these two keywords in their title and are hence not included in this list.

1. arXiv:1306.3610 [pdf, ps, other]
   Thresholds of Spatially Coupled Systems via Lyapunov's Method
   Christian Schlegel, Marat Burnashev
   Comments: 6 pages
   Subjects: **Information Theory (cs.IT)**

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   Memory Efficient Decoders using Spatially Coupled Quasi-Cyclic LDPC Codes
   Vikram Arkalgud Chandrasetty, Sarah J. Johnson, Gottfried Lechner
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   Robust error correction for real-valued signals via message-passing decoding and spatial coupling
   Jean Barbier, Florent Krzakala, Lenka Zdeborová, Pan Zhang
   Comments: 5 pages, 5 figures
   Subjects: **Information Theory (cs.IT)**

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   Rafah El-Khatib, Nicolas Macris, Ruediger Urbanke
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5. arXiv:1303.1858 [pdf, ps, other]
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   Ryunosuke Ohashi, Kenta Kasai, Keigo Takeuchi
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    Louay Bazzi, Badih Ghazi, Rudiger Urbanke
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    Santhosh Kumar, Andrew J. Young, Nicolas Macris, Henry D. Pfister
    Comments: In proceedings of Allerton 2012; Corrected a typo in equation (5)
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    Gianluigi Liva, Enrico Paolini, Michael Lentmaier, Marco Chiani
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    Shrinivas Kudekar, Tom Richardson, Ruediger Urbanke
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    S. Hamed Hassani, Nicolas Macris, Rudiger Urbanke
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