Proposition of semester project (LTHI)

Consider the following process X, with values in $\mathbb{R}^2$: $X_0 = 0$ and then X_{n+1} is chosen uniformly in the disc of center X_n and radius $1 - |X_n|$. The process X is known to be a *martingale*, i.e.

$$\mathbb{E}(X_{n+1}|X_n, \ldots, X_0) = X_n, \quad n \geq 0.$$

Moreover, one can see that X_n stays in the disc of center 0 and radius 1 for all values of n. Therefore, $|X_{n+1} - X_n| \leq 2$ for all n; X is called a martingale "with bounded differences". By a general theorem in probability, such a process always converges to limiting random variable X_∞ as $n \to \infty$ (in the present case, it actually converges to the random variable which is uniformly distributed on the circle of center 0 and radius 1).

The open question is: at what speed does the process converge towards its limit? In order to answer such a question, concentration inequalities will be needed.

Required skills

The student should be at ease with probability and have a taste for theory in general.

Advisor

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