COMPLETELY POSITIVE DYNAMICS OF CORRELATED NEUTRAL KAONS

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Abstract

We study the behaviour of correlated neutral kaons produced in ϕ-meson decays under the hypothesis that the quantum mechanical time-evolution is completely positive. We show that planned experiments at ϕ-factories could give precise bounds on the phenomenological parameters of the model.

1. INTRODUCTION

Recently, it has been proposed to describe the neutral kaon system using effective dynamics not of the standard Weisskopf-Wigner type. These non-standard time-evolutions transform pure states into mixed ones and lead to CP and CPT violating effects [1-4, 5]. The physical motivations behind such an approach are based on quantum gravity, that predicts loss of coherence at the Planck's length due to fluctuations of the gravitational field [6, 7]. Remarkably, the neutral kaon system provides an experimental testing ground for this hypothesis [8] giving bounds to the phenomenological parameters of the theory.

The non-standard quantum dynamics discussed in [1-4, 5] is based on a specific evolution map that acts on the kaon density matrix ρ, $\rho \mapsto \rho(t)$. This map has the property of being (simply) positive, since it preserves the positivity of the eigenvalues of the density matrix $\rho(t)$ for all times $t \geq 0$; it further fulfills other basic physical requirements: it decreases the trace of $\rho(t)$, $\mathrm{d}/\mathrm{dt}\big[\mathrm{Tr}\rho(t)\big] \leq 0$, and increases the entropy, $-\mathrm{d}/\mathrm{dt}\big[\mathrm{Tr}\rho(t)\log\rho(t)\big] \geq 0$. The non-unitarity of this new dynamics is not only due to the decay of the neutral kaons, as in the standard Weisskopf-Wigner phenomenological description, but also to the fact that initially pure kaon states become statistical mixtures.

A different, more general approach is however possible; it treats decay systems as specific examples of open quantum systems.[9-11] These systems can be modeled as being small subsystems in interaction with suitable large environments. The global evolution of the closed compound system is described by an unitary map, while the reduced dynamics of the subsystem usually develops dissipation and irreversibility. Assuming a weak coupling between subsystem and environment, the reduced time-evolution that results from eliminating the environment degrees of freedom is free from non-linear feedback or memory effects, thus possessing very basic and fundamental properties, like forward in time composition (semigroup property), probability conservation and entropy increase. Moreover, the corresponding dynamical map $\gamma_t : \rho \mapsto \rho(t)$ has the additional property of being completely positive [12-15], and not only simply positive. This set of transformations forms a so-called dynamical semigroup.

In the case of the neutral kaons,[16-19] the complete positivity of the time evolution γ_t amounts to the positivity of the natural extension of γ_t to a linear transformation on the states of a larger system consisting of the kaon system and another finite-level system of arbitrary dimension. As we have already remarked, simple positivity guarantees that the eigenvalues of any density matrix for the neutral kaon system remain positive. Complete positivity is a stronger property in the sense that the same holds for density matrices of the compound system.

Although at first sight the requirement of complete positivity instead of the much milder simple positivity seems a mere technical request, it has far-reaching consequences. In particular, when the additional finite-level system alluded above is taken to be another kaon system (a physical situation typically encountered in ϕ-meson decays), the complete positivity of the time-evolution γ_t assures the absence of unphysical effects, like the appearance of negative probabilities, that could occur for simple positive dynamics [18].

In [16, 17] the decay of a single neutral kaon system was studied under the hypothesis of a completely positive non-standard quantum dynamics and the phenomenological parameters specifying the theory were compared with presently available experimental data

[19]. Here, we extend that analysis to the case of correlated neutral kaons under the assumption that, after being produced in a ϕ-meson decay, they evolve independently under the completely positive quantum dynamics. This analysis is motivated by the new experimental facilities that will soon be available for the study of ϕ-meson decays, the so called ϕ-factories. The outcome of our work is that planned experiments at these facilities are likely to put stringent bounds on the phenomenological parameters governing the proposed models.

In Section 2 we describe briefly the dynamics of a single kaon under the hypothesis of complete positivity. This dynamics is then applied in Section 3 to the analysis of the time evolution of correlated kaons, as produced by the decay of a ϕ-meson. As explained in Section 4, this allows a detailed study of the time-evolution of various physical observables that will be experimentally accessible at ϕ-factories. Finally, Section 5 comprises concluding discussions and comments, as well as additional considerations concerning completely positive *vs* simply positive dynamics for correlated kaons.

2. SINGLE KAON COMPLETELY POSITIVE DYNAMICS

As usual, we shall model the evolution and decay of the K^0-$\overline{K^0}$ system by means of a two-dimensional Hilbert space [20]. A convenient orthonormal basis in this space is given by the CP-eigenstates $|K_1\rangle$ and $|K_2\rangle$:

$$|K_1\rangle = \frac{1}{\sqrt{2}}\Big[|K^0\rangle + |\overline{K^0}\rangle\Big]\ , \quad |K_2\rangle = \frac{1}{\sqrt{2}}\Big[|K^0\rangle - |\overline{K^0}\rangle\Big]\ . \tag{2.1}$$

Single kaon states will be described by density matrices ρ, *i.e.* by hermitian 2×2 matrices with positive eigenvalues and unit trace. With respect to the basis (2.1), one can write:

$$\rho = \begin{pmatrix} \rho_1 & \rho_3 \\ \rho_4 & \rho_2 \end{pmatrix}\ , \tag{2.2}$$

where $\rho_4 \equiv \rho_3^*$, and $*$ signifies complex conjugation.

As explained in the Introduction, our analysis is based on the assumption that the evolution in time of a given initial kaon state ρ be completely positive. The corresponding evolution map, $\rho \mapsto \rho(t) \equiv \gamma_t[\rho]$, is then generated by an equation of the following form:

$$\frac{\partial \rho(t)}{\partial t} = -iH\,\rho(t) + i\rho(t)\,H^\dagger + L[\rho]. \tag{2.3}$$

The first two pieces in the r.h.s. constitute the standard Weisskopf-Wigner contribution, while L is a linear map that is fully determined by the request of complete positivity (and trace conservation):

$$L[\rho] = -\frac{1}{2}\sum_j \Big(A_j^\dagger A_j\,\rho + \rho\,A_j^\dagger A_j\Big)\ + \sum_j A_j\,\rho\,A_j^\dagger\ . \tag{2.4}$$

The operators A_j must be such that $\sum_j A_j^\dagger A_j$ is a well-defined 2×2 matrix; further, to assure entropy increase, the A_j will be taken to be hermitian.

In absence of $L[\rho]$, pure states (*i.e.* states of the form $|\psi\rangle\langle\psi|$) would be transformed into pure states, in spite of the fact that probability is not conserved. However, this is due to the presence of a non-hermitian part in the effective hamiltonian H and not to a mixing-enhancing mechanism that makes $\rho(t)$ less ordered in time. Instead, loss of quantum coherence shows up when the extra piece $L[\rho]$ is also present: it produces dissipation and possible transitions from pure states to mixed states.

The effective hamiltonian H (the Weisskopf-Wigner hamiltonian) includes a nonhermitian part, that characterizes the natural width of the states:

$$H = M - \frac{i}{2}\mathit{\Gamma}\ , \tag{2.5}$$

with M and $\mathit{\Gamma}$ hermitian 2×2 matrices. The entries of these matrices can be expressed in terms of the four real parameters, m_S, γ_S and m_L, γ_L characterizing the eigenvalues of H:

$$\lambda_S = m_S - \frac{i}{2}\gamma_S\ ,\quad \lambda_L = m_L - \frac{i}{2}\gamma_L\ , \tag{2.6}$$

and the two complex parameters ϵ_S, ϵ_L, appearing in the eigenstates of H,

$$\begin{aligned} |K_S\rangle &= N_S\Big(|K_1\rangle\ +\ \epsilon_S|K_2\rangle\Big)\\ |K_L\rangle &= N_L\Big(\epsilon_L|K_1\rangle\ +\ |K_2\rangle\Big)\ , \end{aligned} \tag{2.7}$$

with $N_S = (1+|\epsilon_S|^2)^{-1/2}$ and $N_L = (1+|\epsilon_L|^2)^{-1/2}$ normalization factors. It proves convenient to use also the following positive combinations:

$$\Delta\Gamma = \gamma_S - \gamma_L\ ,\qquad \Delta m = m_L - m_S\ , \tag{2.8}$$

corresponding to the differences between decay widths and masses of the states K_S and K_L, as well as of the complex quantities:

$$\Gamma_\pm = \Gamma \pm i\Delta m\ ,\qquad \Delta\Gamma_\pm = \Delta\Gamma \pm 2i\Delta m \tag{2.9}$$

with $\Gamma = (\gamma_S+\gamma_L)/2$.

The explicit form of the piece $L[\rho]$ can be most simply given by expanding ρ in terms of Pauli matrices σ_i and the identity σ_0: $\rho = \rho_\mu\,\sigma_\mu$, $\mu =$ 0, 1, 2, 3. In this way, the map $L[\rho]$ can be represented by a symmetric 4×4 matrix $\big[L_{\mu\nu}\big]$, acting on the column vector with components $(\rho_0,\rho_1,\rho_2,\rho_3)$. It can be parametrized by the six real constants a, b, c, α, β, and γ: [16, 17]

$$\big[L_{\mu\nu}\big] = -2\begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & a & b & c\\ 0 & b & \alpha & \beta\\ 0 & c & \beta & \gamma \end{pmatrix}\ , \tag{2.10}$$

with a, α and γ non-negative. These parameters are not all independent; to assure the complete positivity of the time-evolution $\rho \to \rho(t)$, they have to satisfy the following inequalities:

$$\begin{aligned} &a \leq \alpha+\gamma\ , \qquad && 4b^2 \leq \gamma^2-\big(a-\alpha\big)^2\ ,\\ &\alpha \leq a+\gamma\ , && 4c^2 \leq \alpha^2-\big(a-\gamma\big)^2\ ,\\ &\gamma \leq a+\alpha\ , && 4\beta^2 \leq a^2-\big(\alpha-\gamma\big)^2\ . \end{aligned} \tag{2.11}$$

Physical observables of the neutral kaon system can be obtained from the density matrix $\rho(t)$ obeying (2.3) by taking its trace with suitable hermitian operators. In order to study the time evolution of these observables, one has to solve the equation (2.3) for a given initial state ρ, *i.e.* compute the entries of the 4×4 evolution matrix $M_{ij}(t)$, which in the basis (2.1), (2.2) gives the entries of the density matrix at time t:

$$\rho_i(t)=\sum_{j=1}^{4} M_{ij}(t)\,\rho_j\ ,\qquad i=1,2,3,4\ . \tag{2.12}$$

According to plausible phenomenological considerations,[1-4, 5, 17] the parameters a, b, c, α, β and γ can be assumed to be small, of the same order of magnitude of $\epsilon_S\Delta\Gamma$ and $\epsilon_L\Delta\Gamma$; the use of perturbation theory is therefore justified. For most purposes, it is sufficient to consider contributions up to the second order in these small parameters. The expansion of $M_{ij}(t)$ within this approximation can be conveniently organized as the sum of three contributions:[17]

$$\rho_i(t)\simeq\sum_{j=1}^{4}\Big[M_{ij}^{(0)}(t)\ +\ M_{ij}^{(1)}(t)\ +\ M_{ij}^{(2)}(t)\Big]\,\rho_j\ . \tag{2.13}$$

For further reference, we list below the explicit expressions of the entries in the matrices $M^{(0)}$ and $M^{(1)}$. The contributions of $M^{(2)}$ can be found in [17], together with a detailed discussion of the method used in their derivation. The matrix $M^{(0)}$ has only diagonal non-vanishing entries:

$$M_{11}^{(0)}(t)=e^{-\gamma_S t}\ ,\quad M_{22}^{(0)}(t)=e^{-\gamma_L t}\ ,\quad M_{33}^{(0)}(t)=e^{-\widetilde\Gamma_- t}\ ,\quad M_{44}^{(0)}(t)=e^{-\widetilde\Gamma_+ t}\ , \tag{2.14}$$

where

$$\widetilde\Gamma_\pm=\Gamma_\pm+A-\gamma-2\,\mathcal{R}e\,[C(\epsilon_S^*-\epsilon_L)]\mp i\frac{|B|^2}{2\Delta m}\mp 8i\,\Delta m\left|\frac{C}{\Delta\Gamma_+}\right|^2\ , \tag{2.15}$$

and A, B, C are the following convenient combinations of the parameters in (2.10):

$$A=\alpha+a\ ,\quad B=\alpha-a+2ib\ ,\quad C=c+i\beta\ . \tag{2.16}$$

The entries of $M^{(1)}$ take instead the following explicit expression:

$$
\begin{aligned}
&M^{(1)}_{11}(t) = 0 && M^{(1)}_{12}(t) = \frac{\gamma}{\Delta\Gamma}\Big(e^{-\gamma_L t} - e^{-\gamma_S t}\Big)\\
&M^{(1)}_{13}(t) = e^*_L(-,-)\Big(e^{-(\Gamma_-+A-\gamma)t} - e^{-\gamma_S t}\Big) && M^{(1)}_{14}(t) = e_L(-,-)\Big(e^{-(\Gamma_++A-\gamma)t} - e^{-\gamma_S t}\Big)\\
&M^{(1)}_{21}(t) = \frac{\gamma}{\Delta\Gamma}\Big(e^{-\gamma_L t} - e^{-\gamma_S t}\Big) && M^{(1)}_{22}(t) = 0\\
&M^{(1)}_{23}(t) = f_S(-,-)\Big(e^{-(\Gamma_-+A-\gamma)t} - e^{-\gamma_L t}\Big) && M^{(1)}_{24}(t) = f^*_S(-,-)\Big(e^{-(\Gamma_++A-\gamma)t} - e^{-\gamma_L t}\Big)\\
&M^{(1)}_{31}(t) = f^*_S(+,-)\Big(e^{-\gamma_S t} - e^{-(\Gamma_-+A-\gamma)t}\Big) && M^{(1)}_{32}(t) = e_L(+,-)\Big(e^{-\gamma_L t} - e^{-(\Gamma_-+A-\gamma)t}\Big)\\
&M^{(1)}_{33}(t) = 0 && M^{(1)}_{34}(t) = \frac{iB}{2\Delta m}\, e^{-(A-\gamma)t}\Big(e^{-\Gamma_+ t} - e^{-\Gamma_- t}\Big)\\
&M^{(1)}_{41}(t) = f_S(+,-)\Big(e^{-\gamma_S t} - e^{-(\Gamma_++A-\gamma)t}\Big) && M^{(1)}_{42}(t) = e^*_L(+,-)\Big(e^{-\gamma_L t} - e^{-(\Gamma_++A-\gamma)t}\Big)\\
&M^{(1)}_{43}(t) = \frac{iB^*}{2\Delta m}\, e^{-(A-\gamma)t}\Big(e^{-\Gamma_+ t} - e^{-\Gamma_- t}\Big) && M^{(1)}_{44}(t) = 0\ ,
\end{aligned}
\tag{2.17}
$$

where we have introduced the shorthand notations:

$$
\begin{aligned}
&e_{S,L}(+,\pm) = \epsilon_{S,L} + \frac{2C^*}{\Delta\Gamma_\pm}\ , \qquad && f_{S,L}(+,\pm) = \epsilon_{S,L} + \frac{2C}{\Delta\Gamma_\pm}\ ,\\
&e_{S,L}(-,\pm) = \epsilon_{S,L} - \frac{2C^*}{\Delta\Gamma_\pm}\ , \qquad && f_{S,L}(-,\pm) = \epsilon_{S,L} - \frac{2C}{\Delta\Gamma_\pm}\ .
\end{aligned}
\tag{2.18}
$$

Notice that in presenting the above expressions for the entries of $M^{(0)}$ and $M^{(1)}$, we have reconstructed the exponential dependences out of first and second order correction terms which are linear and quadratic in time. To a given order in the perturbative expansion, this can always be done provided one suitably redefines $\Gamma_\pm$ and $\Delta\Gamma_\pm$ in (2.9) in such a way that γ_S, γ_L and Δm can be directly identified with the widths and mass-difference of the K_S and K_L physical states (for more details, see the discussion in the Appendix of [17]).

In [17, 19] the approximate solution (2.13) to the evolution equation (2.3) has been used to compute the time dependence of many physical observables of the single kaon system relevant to the experiment. In the next section, we shall apply it to the study of the time evolution of correlated systems of two neutral kaons.

3. COMPLETELY POSITIVE DYNAMICS OF CORRELATED KAONS

The time evolution of a system of two correlated neutral kaons can be discussed using the results on the dynamics of a single kaon of the previous section. We shall limit the analysis to the study of entangled K^0-$\overline{K^0}$ states coming from the decay of a ϕ-meson. This is of great experimental relevance in view of the possibilities offered by ϕ-factories.

Since the ϕ-meson has spin 1, its decay into two spinless bosons produces an antisymmetric spatial state. In the ϕ rest frame, the two neutral kaons are produced flying apart with opposite momenta; in the basis $|K_1\rangle$, $|K_2\rangle$, the resulting state can be described by:

$$|\psi_A\rangle = \frac{1}{\sqrt{2}}\Big(|K_1, -p\rangle \otimes |K_2, p\rangle - |K_2, -p\rangle \otimes |K_1, p\rangle\Big)\ . \tag{3.1}$$

The corresponding density operator ρ_A is a 4×4 matrix that can be conveniently written in terms of single kaon projectors:

$$P_1 \equiv |K_1\rangle\langle K_1| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\ , \qquad P_2 \equiv |K_2\rangle\langle K_2| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\ , \tag{3.2a}$$

and the off-diagonal operators

$$P_3 \equiv |K_1\rangle\langle K_2| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\ , \qquad P_4 \equiv |K_2\rangle\langle K_1| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\ . \tag{3.2b}$$

Explicitly, one finds:

$$\rho_A = \frac{1}{2}\Big[P_1 \otimes P_2\ +\ P_2 \otimes P_1\ -\ P_3 \otimes P_4\ -\ P_4 \otimes P_3\Big]\ . \tag{3.3}$$

In studying the dynamics of correlated kaons, we shall assume that, once produced in a ϕ decay, the kaons evolve in time each according to the completely positive map γ_t described in the previous section. This assures that the resulting evolution map $\rho_A \mapsto \Gamma_t[\rho_A]$ is completely positive and of semigroup type; further, this dynamics is independent from the particular situation under study and can be easily generalized to systems containing more than two particles. Although other possibilities are conceivable, this choice is the most natural one. In fact, it is very hard to produce dynamical maps Γ_t for a system of two particles not in factorized form without violating very basic physical principles. Indeed, if one requires that after tracing over the degrees of freedom of one particle, the resulting dynamics for the remaining one be a) completely positive, b) of semigroup type and c) independent from the initial state of the first particle, than the only natural possibility is: $\Gamma_t = \gamma_t \otimes \gamma_t$.

With this choice, the density matrix that describes a situation in which the first kaon has evolved up to proper time τ_1 and the second up to proper time τ_2 is given by:

$$\begin{aligned}\rho_A(\tau_1, \tau_2) &\equiv \big(\gamma_{\tau_1} \otimes \gamma_{\tau_2}\big)\big[\rho_A\big] \\ &= \frac{1}{2}\Big[P_1(\tau_1) \otimes P_2(\tau_2)\ +\ P_2(\tau_1) \otimes P_1(\tau_2)\ - P_3(\tau_1) \otimes P_4(\tau_2) - P_4(\tau_1) \otimes P_3(\tau_2)\Big]\ ,\end{aligned} \tag{3.4}$$

where $P_i(\tau_1)$ and $P_i(\tau_2)$, $i = 1, 2, 3, 4$, represent the evolution according to (2.3) of the initial operators (3.2), up to the time τ_1 and τ_2, respectively.

As already noticed, the system of the two neutral kaons that is produced in the decay of a ϕ-meson is highly correlated and therefore constitutes a unique setup for studying phenomena involving loss of quantum coherence. The typical observables that can be studied in such quantum interferometer are double decay rates, *i.e.* the probabilities that a kaon decays into a final state f_1 at proper time τ_1, while the other kaon decays into the final state f_2 at proper time τ_2.[21-25] Such correlations will be denoted by $\mathcal{G}(f_1,\tau_1;f_2,\tau_2)$. Using (3.4), one explicitly finds:

$$
\begin{aligned}
\mathcal{G}(f_1,\tau_1;&f_2,\tau_2)\equiv \mathrm{Tr}\Big[\Big(\mathcal{O}_{f_1}\otimes\mathcal{O}_{f_2}\Big)\rho_A(\tau_1,\tau_2)\Big]\\
&=\frac{1}{2}\Big[\mathrm{Tr}\big\{P_1(\tau_1)\,\mathcal{O}_{f_1}\big\}\ \mathrm{Tr}\big\{P_2(\tau_2)\,\mathcal{O}_{f_2}\big\}\ +\ \mathrm{Tr}\big\{P_2(\tau_1)\,\mathcal{O}_{f_1}\big\}\ \mathrm{Tr}\big\{P_1(\tau_2)\,\mathcal{O}_{f_2}\big\}\\
&-\mathrm{Tr}\big\{P_3(\tau_1)\,\mathcal{O}_{f_1}\big\}\ \mathrm{Tr}\big\{P_4(\tau_2)\,\mathcal{O}_{f_2}\big\}\ -\ \mathrm{Tr}\big\{P_4(\tau_1)\,\mathcal{O}_{f_1}\big\}\ \mathrm{Tr}\big\{P_3(\tau_2)\,\mathcal{O}_{f_2}\big\}\Big]\ .
\end{aligned}
\tag{3.5}
$$

In this formula, $\mathcal{O}_{f_1}$ and $\mathcal{O}_{f_2}$ represent the 2×2 hermitian matrices describing the decay of a single kaon into the final states f_1 and f_2, respectively. Any operator $\mathcal{O}$ of such type can be written in the basis (2.1) using the the matrices P_i, $i=1,2,3,4$ of (3.2):

$$
\mathcal{O}=\sum_{i=1}^{4}\mathcal{O}^i\ P_i\ .
\tag{3.6}
$$

Therefore, the observables (3.5) can be rewritten as

$$
\mathcal{G}(f_1,\tau_1;f_2,\tau_2)=\frac{1}{2}\sum_{i,j=1}^{4}\mathcal{O}^i_{f_1}\,\mathcal{O}^j_{f_2}\ \mathcal{P}_{ij}(\tau_1,\tau_2)\ ,
\tag{3.7}
$$

where

$$
\begin{aligned}
\mathcal{P}_{ij}(\tau_1,\tau_2)=\mathrm{Tr}\big\{P_i\,P_1(\tau_1)\big\}\ \mathrm{Tr}\big\{P_j\,P_2(\tau_2)\big\}\ +\ \mathrm{Tr}\big\{P_i\,P_2(\tau_1)\big\}\ \mathrm{Tr}\big\{P_j\,P_1(\tau_2)\big\}\\
-\ \mathrm{Tr}\big\{P_i\,P_3(\tau_1)\big\}\ \mathrm{Tr}\big\{P_j\,P_4(\tau_2)\big\}\ -\ \mathrm{Tr}\big\{P_i\,P_4(\tau_1)\big\}\ \mathrm{Tr}\big\{P_j\,P_3(\tau_2)\big\}\ .
\end{aligned}
\tag{3.8}
$$

These "elementary" double probabilities $\mathcal{P}_{ij}$ can be computed in perturbation theory using the solution to the evolution equation (2.3) discussed in Section 2. Expansions for $\mathcal{P}_{ij}$ up to second order in the small parameters will suffice to produce accurate enough expressions for the correlations $\mathcal{G}$ that will be examined in the following. The explicit expressions of the various probabilities $\mathcal{P}_{ij}$ to this degree of accuracy are collected in Appendix A. One finds that $\mathcal{P}_{12}$, $\mathcal{P}_{21}$, $\mathcal{P}_{34}$ and $\mathcal{P}_{43}$ possess zero-th and second order contributions in the small parameters, but not first order terms. All the other probabilities $\mathcal{P}_{ij}$ have only first and second order contributions.

As we shall see in the next section, the completely positive dynamics (3.4) produces results for the double probabilities $\mathcal{G}(f_1,\tau_1;f_2,\tau_2)$ that substantially differ from the ones obtained using standard quantum mechanics. The most striking difference arises in considering correlations at equal proper times $\tau_1=\tau_2=\tau$, with the same final state $f_1=f_2=f$. Due to the antisymmetry of the initial state ρ_A in (3.3), ordinary quantum mechanics predicts a vanishing result for $\mathcal{G}(f,\tau;f,\tau)$. This is in general not the case for the completely positive dynamics discussed here. The equal-time correlations $\mathcal{G}(f,\tau;f,\tau)$ are therefore very sensible to the non-standard parameters a, b, c, α, β and γ. A direct experimental determination of the probabilities $\mathcal{G}(f,\tau;f,\tau)$ is clearly problematic; however, these correlations can be extrapolated from the measured double-time ones, $\mathcal{G}(f,\tau_1;f,\tau_2)$, when the interval $\tau_1-\tau_2$ becomes small. Therefore, the correlations $\mathcal{G}(f,\tau;f,\tau)$ can be very useful for obtaining experimental estimates for the parameters a, b, c, α, β and γ.

4. OBSERVABLES

A high-luminosity ϕ-factory has been recognized as one of the best experimental setup for studying CP and CPT violating effects.[21-25] The machine, producing ϕ-mesons at high rate, behaves as a very accurate quantum interferometer capable of revealing tiny effects and of measuring very small quantities. Therefore, it is the natural experimental setup for measuring probabilities involving double kaon decays of the type introduced in the previous section. In the discussion that follows, we shall ignore effects due to finite detector size and to background processes that produce couple of K^0-$\overline{K^0}$ in a symmetric spin zero combinations. We shall comment on some of these background effects in the final section.

In order to compute the correlations $\mathcal{G}(f_1,\tau_1;f_2,\tau_2)$, one needs to give explicit expressions for the matrices $\mathcal{O}_f$ that describe the decay of a single kaon into the final state f. Useful observables $\mathcal{G}$ are associated with the decays of the neutral kaons into two and three pions and into semileptonic states. We shall be as general as possible and use matrices $\mathcal{O}_f$ that encode possible CP and CPT violating effects also in the decay amplitudes.

In the case of the $\pi^+\pi^-$ and $\pi^0\pi^0$ final states, using the basis K_1, K_2 of (2.1), we shall label the corresponding decay amplitudes as:

$$\begin{aligned} &\mathcal{A}(K_1\to\pi^+\pi^-)=X_{+-}\ , \qquad &&\mathcal{A}(K_2\to\pi^+\pi^-)=Y_{+-}\ \mathcal{A}(K_1\to\pi^+\pi^-)\ ,\\ &\mathcal{A}(K_1\to\pi^0\pi^0)=X_{00}\ , &&\mathcal{A}(K_2\to\pi^0\pi^0)=Y_{00}\ \mathcal{A}(K_1\to\pi^0\pi^0)\ . \end{aligned} \tag{4.1}$$

The operators $\mathcal{O}$ that describe these final states take then the following form [5, 19]:

$$\mathcal{O}_{+-}=|X_{+-}|^2\begin{bmatrix}1 & Y_{+-}\\ Y_{+-}^* & |Y_{+-}|^2\end{bmatrix}\ ,\qquad \mathcal{O}_{00}=|X_{00}|^2\begin{bmatrix}1 & Y_{00}\\ Y_{00}^* & |Y_{00}|^2\end{bmatrix}\ . \tag{4.2}$$

Similar expressions hold for the decay into three pions, $\pi^+\pi^-\pi^0$ (with total isospin $I=1$ [20]) and $\pi^0\pi^0\pi^0$. Parametrizing the decay amplitudes as

$$\begin{aligned} &\mathcal{A}(K_2\to\pi^+\pi^-\pi^0)=X_{+-0}\ , \qquad &&\mathcal{A}(K_1\to\pi^+\pi^-\pi^0)=Y_{+-0}\ \mathcal{A}(K_2\to\pi^+\pi^-\pi^0)\ ,\\ &\mathcal{A}(K_2\to\pi^0\pi^0\pi^0)=X_{000}\ , &&\mathcal{A}(K_1\to\pi^0\pi^0\pi^0)=Y_{000}\ \mathcal{A}(K_2\to\pi^0\pi^0\pi^0)\ , \end{aligned} \tag{4.3}$$

one finds [5, 19]

$$\mathcal{O}_{+-0}=|X_{+-0}|^2\begin{bmatrix}|Y_{+-0}|^2 & Y_{+-0}^*\\ Y_{+-0} & 1\end{bmatrix}\ ,\qquad \mathcal{O}_{000}=|X_{000}|^2\begin{bmatrix}|Y_{000}|^2 & Y_{000}^*\\ Y_{000} & 1\end{bmatrix}\ . \tag{4.4}$$

The parameters Y in (4.1) and (4.3), when expressed in terms of the K^0, $\overline{K^0}$ amplitudes, are seen to depend linearly on CP and CPT violating terms. Therefore, in the following they will be treated as the other small parameters of the theory; in particular, terms containing these parameters to a degree higher than two will be dropped.

With the help of the expressions (4.2) and (4.4) one can now compute correlations $\mathcal{G}$ involving pions in the final states. Keeping only up to second order terms in all small parameters a, b, c, α, β, γ and Y's, one finds:

$$\begin{aligned}\mathcal{G}(\pi^+\pi^-,\tau_1;\pi^+\pi^-,\tau_2) = \frac{|X_{+-}|^4}{2}\Big\{&\mathcal{P}_{11}(\tau_1,\tau_2) + |Y_{+-}|^2\big[\mathcal{P}_{12}(\tau_1,\tau_2)+\mathcal{P}_{21}(\tau_1,\tau_2)\big]\\ &+ 2|Y_{+-}|^2\,\mathcal{R}e\big[\mathcal{P}_{34}(\tau_1,\tau_2)\big] + 2\,\mathcal{R}e\Big(Y_{+-}\big[\mathcal{P}_{13}(\tau_1,\tau_2)+\mathcal{P}_{31}(\tau_1,\tau_2)\big]\Big)\Big\}\ , \end{aligned}\tag{4.5a}$$

$$\begin{aligned}\mathcal{G}(\pi^+\pi^-,\tau_1;2\pi^0,\tau_2) = \frac{|X_{+-}|^2|X_{00}|^2}{2}\Big\{&\mathcal{P}_{11}(\tau_1,\tau_2) + |Y_{00}|^2\mathcal{P}_{12}(\tau_1,\tau_2) + |Y_{+-}|^2\mathcal{P}_{21}(\tau_1,\tau_2)\\ &+ 2\,\mathcal{R}e\Big[Y_{00}\,\mathcal{P}_{13}(\tau_1,\tau_2) + Y_{+-}\,\mathcal{P}_{31}(\tau_1,\tau_2) + Y_{+-}Y^*_{00}\,\mathcal{P}_{34}(\tau_1,\tau_2)\Big]\Big\}\ , \end{aligned}\tag{4.5b}$$

$$\begin{aligned}\mathcal{G}(\pi^+\pi^-\pi^0,\tau_1;\pi^+\pi^-\pi^0,\tau_2) = \frac{|X_{+-0}|^4}{2}\Big\{&\mathcal{P}_{22}(\tau_1,\tau_2) + |Y_{+-0}|^2\big[\mathcal{P}_{12}(\tau_1,\tau_2)+\mathcal{P}_{21}(\tau_1,\tau_2)\big]\\ + 2|Y_{+-0}|^2\,\mathcal{R}e\big[\mathcal{P}_{34}(\tau_1,\tau_2)\big] &+ 2\,\mathcal{R}e\Big(Y^*_{+-0}\big[\mathcal{P}_{23}(\tau_1,\tau_2)+\mathcal{P}_{32}(\tau_1,\tau_2)\big]\Big)\Big\}\ , \end{aligned}\tag{4.5c}$$

$$\begin{aligned}&\mathcal{G}(\pi^+\pi^-\pi^0,\tau_1;3\pi^0,\tau_2) = \frac{|X_{+-0}|^2|X_{000}|^2}{2}\Big\{\mathcal{P}_{22}(\tau_1,\tau_2) + |Y_{+-0}|^2\mathcal{P}_{12}(\tau_1,\tau_2)\\ &+ |Y_{000}|^2\mathcal{P}_{21}(\tau_1,\tau_2) + 2\,\mathcal{R}e\Big[Y^*_{+-0}\,\mathcal{P}_{32}(\tau_1,\tau_2) + Y^*_{000}\,\mathcal{P}_{23}(\tau_1,\tau_2) + Y^*_{+-0}Y_{000}\,\mathcal{P}_{34}(\tau_1,\tau_2)\Big]\Big\}\ . \end{aligned}\tag{4.5d}$$

Explicit expressions for all these quantities can be found in Appendix B. Other correlations are $\mathcal{G}(2\pi^0,\tau_1;\pi^+\pi^-,\tau_2)$ and $\mathcal{G}(3\pi^0,\tau_1;\pi^+\pi^-\pi^0,\tau_2)$; they can be obtained from (4.5*b*) and (4.5*d*) by formally exchanging τ_1 and τ_2. Further, the probabilities $\mathcal{G}(2\pi^0,\tau_1;2\pi^0\tau_2)$ and $\mathcal{G}(3\pi^0,\tau_1;3\pi^0,\tau_2)$, can be obtained from (4.5*a*) and (4.5*c*) by replacing X_{+-}, Y_{+-} with X_{00}, Y_{00} and X_{+-0}, Y_{+-0} with X_{000}, Y_{000}, respectively.

As already observed at the end of the previous section, it is of great interest to study correlations $\mathcal{G}$ at equal proper times, in particular when the two final states coincide. These correlations are all proportional to the non-standard parameters a, b, c, α, β, γ and can be used to obtain experimental estimates on these constants. In the case of the $\pi^+\pi^-$ final states, one finds:

$$\begin{aligned}\mathcal{G}(\pi^+\pi^-,\tau;\pi^+\pi^-,\tau) = |X_{+-}|^4\,e^{-\gamma_S\tau}\Big\{&e^{-\gamma_L\tau}\Big(R^L_{+-} - |\eta_{+-}|^2\Big)\\ - e^{-\gamma_S\tau}\left[\frac{\gamma}{\Delta\Gamma} + 8\left|\frac{C}{\Delta\Gamma_+}\right|^2 - 4\,\mathcal{R}e\left(\frac{\epsilon_L\,C}{\Delta\Gamma}\right)\right] &- e^{-\Gamma\tau}\,8\,\mathcal{R}e\left(\frac{\eta_{+-}\,C}{\Delta\Gamma_+}e^{-i\Delta m\tau}\right)\Big\}\ , \end{aligned}\tag{4.6}$$

where

$$R^L_{+-} = \left|\epsilon_L + \frac{2C^*}{\Delta\Gamma_-} + Y_{+-}\right|^2 + \frac{\gamma}{\Delta\Gamma} - 8\left|\frac{C}{\Delta\Gamma_+}\right|^2 - 4\,\mathcal{R}e\left(\frac{\epsilon_L C}{\Delta\Gamma}\right)\ ,\tag{4.7a}$$

is the $\pi^+\pi^-$ decay rate for the K_L state, while

$$\eta_{+-} = \epsilon_L - \frac{2C^*}{\Delta\Gamma_-} + Y_{+-}\ , \tag{4.7b}$$

is the parameter that measures the K_L, K_S amplitudes ratio for the $\pi^+\pi^-$ decay. Similar results hold for the $\pi^+\pi^-\pi^0$-decay; explicitly, one gets:

$$\mathcal{G}(\pi^+\pi^-\pi^0,\tau;\pi^+\pi^-\pi^0,\tau) = |X_{+-0}|^4\, e^{-\gamma_L\tau}\bigg\{ e^{-\gamma_S\tau}\Big(R^S_{+-0} - |\eta_{+-0}|^2\Big)$$
$$+ e^{-\gamma_L\tau}\left[\frac{\gamma}{\Delta\Gamma} - 8\left|\frac{C}{\Delta\Gamma_+}\right|^2 + 4\,\mathcal{R}e\bigg(\frac{\epsilon_S\, C^*}{\Delta\Gamma}\bigg)\right] - e^{-\Gamma\tau}\, 8\,\mathcal{R}e\bigg(\frac{\eta_{+-0}\, C^*}{\Delta\Gamma_+} e^{i\Delta m\tau}\bigg)\bigg\}\ , \tag{4.8}$$

where

$$R^S_{+-0} = \left|\epsilon_S + \frac{2C}{\Delta\Gamma_-} + Y_{+-0}\right|^2 - \frac{\gamma}{\Delta\Gamma} - 8\left|\frac{C}{\Delta\Gamma_+}\right|^2 - 4\,\mathcal{R}e\bigg(\frac{\epsilon_S C^*}{\Delta\Gamma}\bigg) \tag{4.9a}$$

is the CP-violating $\pi^+\pi^-\pi^0$-decay rate of the K_S state and

$$\eta_{+-0} = \epsilon_S - \frac{2C}{\Delta\Gamma_-} + Y_{+-0}\ , \tag{4.9b}$$

measures the ratio between the $\pi^+\pi^-\pi^0$-decay amplitudes for the K_S (with isospin $I=1$) and K_L states.[26] In particular, notice that the long time behaviour ($\tau \gg 1/\gamma_S$) of the probability (4.8) can give direct informations on the parameter γ:

$$\mathcal{G}(\pi^+\pi^-\pi^0,\tau\,;\pi^+\pi^-\pi^0,\tau) \sim \frac{\gamma}{\Delta\Gamma}\ e^{-2\gamma_L\tau}\ . \tag{4.10}$$

Correlations among decays into semileptonic states are also of great interest. The amplitudes for the decay of a K^0 or a $\overline{K^0}$ state into $\pi^-\ell^+\nu$ and $\pi^+\ell^-\bar\nu$ are usually parametrized by three complex constants x, y and z as follows [27]:

$$\mathcal{A}(K^0 \to \pi^-\ell^+\nu) = \mathcal{M}(1-y)\ , \tag{4.11a}$$
$$\mathcal{A}(\overline{K^0} \to \pi^+\ell^-\bar\nu) = \mathcal{M}^*(1+y^*)\ , \tag{4.11b}$$
$$\mathcal{A}(K^0 \to \pi^+\ell^-\bar\nu) = z\,\mathcal{A}(\overline{K^0} \to \pi^+\ell^-\bar\nu)\ , \tag{4.11c}$$
$$\mathcal{A}(\overline{K^0} \to \pi^-\ell^+\nu) = x\,\mathcal{A}(K^0 \to \pi^-\ell^+\nu)\ , \tag{4.11d}$$

where $\mathcal{M}$ is a common factor. (In [27] $\bar x \equiv z^*$ is used instead of z.) The $\Delta S = \Delta Q$ rule would forbid the decays $K^0 \to \pi^+\ell^-\bar\nu$ and $\overline{K^0} \to \pi^-\ell^+\nu$, so that the parameters x and z measure the violations of this rule. Instead, CPT-invariance would require $y=0$.

From the above parametrization, one derive the decay amplitudes for the states (2.1) of definite CP, and therefore the following expressions for the two operators describing the semileptonic decays:[5, 19]

$$\mathcal{O}_{\ell^+} = \frac{|\mathcal{M}|^2}{2}\,|1-y|^2\,\begin{bmatrix} |1+x|^2 & (1+x^*)(1-x) \\ (1+x)(1-x^*) & |1-x|^2 \end{bmatrix}\ , \tag{4.12a}$$

$$\mathcal{O}_{\ell^-} = \frac{|\mathcal{M}|^2}{2}\,|1+y|^2\,\begin{bmatrix} |z+1|^2 & (z^*+1)(z-1) \\ (z+1)(z^*-1) & |z-1|^2 \end{bmatrix}\ . \tag{4.12b}$$

The parameters x, y, and z are expected to be very small, and the available experimental determinations support this theoretical hypothesis. Therefore, in the following we shall treat these constants as the other small parameters in the theory. While the pion correlations (4.5) require contributions up to second order for comparison with the standard quantum mechanical expressions, in the case of semileptonic decays we can limit the discussion to the contributions up to first order in all small parameters. Taking into account this approximation, from (3.5) and (4.12), the semileptonic double probabilities take the form:

$$\begin{aligned}\mathcal{G}(\ell^+,\tau_1;\ell^-,\tau_2) = \frac{|\mathcal{M}|^4}{4}\Big\{&\mathcal{P}_{11}(\tau_1,\tau_2)+\mathcal{P}_{22}(\tau_1,\tau_2)+\mathcal{P}_{12}(\tau_1,\tau_2)+\mathcal{P}_{21}(\tau_1,\tau_2)\\
&+2\,\mathcal{R}e(x-z)\Big[\mathcal{P}_{12}(\tau_1,\tau_2)-\mathcal{P}_{21}(\tau_1,\tau_2)\Big]\\
&-2\,\mathcal{R}e\Big[\mathcal{P}_{13}(\tau_1,\tau_2)-\mathcal{P}_{31}(\tau_1,\tau_2)+\mathcal{P}_{23}(\tau_1,\tau_2)-\mathcal{P}_{32}(\tau_1,\tau_2)\Big]\\
&-2\,\mathcal{R}e\Big[\mathcal{P}_{33}(\tau_1,\tau_2)+\big[1-2i\mathcal{I}m(x-z)\big]\mathcal{P}_{34}(\tau_1,\tau_2)\Big]\Big\}\end{aligned} \tag{4.13a}$$

$$\begin{aligned}\mathcal{G}(\ell^+,\tau_1;\ell^+,\tau_2) = \frac{|\mathcal{M}|^4}{4}\Big\{&\mathcal{P}_{11}(\tau_1,\tau_2)+\mathcal{P}_{22}(\tau_1,\tau_2)+\mathcal{P}_{12}(\tau_1,\tau_2)+\mathcal{P}_{21}(\tau_1,\tau_2)\\
&-4\,\mathcal{R}e(y)\Big[\mathcal{P}_{12}(\tau_1,\tau_2)+\mathcal{P}_{21}(\tau_1,\tau_2)\Big]\\
&+2\,\mathcal{R}e\Big[\mathcal{P}_{13}(\tau_1,\tau_2)+\mathcal{P}_{31}(\tau_1,\tau_2)+\mathcal{P}_{23}(\tau_1,\tau_2)+\mathcal{P}_{32}(\tau_1,\tau_2)\Big]\\
&+2\,\mathcal{R}e\Big[\mathcal{P}_{33}(\tau_1,\tau_2)+\big[1-4\mathcal{R}e(y)\big]\mathcal{P}_{34}(\tau_1,\tau_2)\Big]\Big\}\end{aligned} \tag{4.13b}$$

$$\begin{aligned}\mathcal{G}(\ell^-,\tau_1;\ell^-,\tau_2) = \frac{|\mathcal{M}|^4}{4}\Big\{&\mathcal{P}_{11}(\tau_1,\tau_2)+\mathcal{P}_{22}(\tau_1,\tau_2)+\mathcal{P}_{12}(\tau_1,\tau_2)+\mathcal{P}_{21}(\tau_1,\tau_2)\\
&+4\,\mathcal{R}e(y)\Big[\mathcal{P}_{12}(\tau_1,\tau_2)+\mathcal{P}_{21}(\tau_1,\tau_2)\Big]\\
&-2\,\mathcal{R}e\Big[\mathcal{P}_{13}(\tau_1,\tau_2)+\mathcal{P}_{31}(\tau_1,\tau_2)+\mathcal{P}_{23}(\tau_1,\tau_2)+\mathcal{P}_{32}(\tau_1,\tau_2)\Big]\\
&+2\,\mathcal{R}e\Big[\mathcal{P}_{33}(\tau_1,\tau_2)+\big[1+4\mathcal{R}e(y)\big]\mathcal{P}_{34}(\tau_1,\tau_2)\Big]\Big\}\ .\end{aligned} \tag{4.13c}$$

Explicit expressions for these correlations are collected in Appendix B. Finally, notice that $\mathcal{G}(\ell^-,\tau_1;\,\ell^+,\tau_2)$ can be formally obtained from (4.13a) by exchanging τ_1 and τ_2.

As in the case of the pion correlations, the equal-time version of the above double probabilities give direct information on the non-standard parameters a, b, c, α, β and γ. Indeed, one finds:

$$\begin{aligned}\mathcal{G}(\ell^\pm,\tau;\ell^\pm,\tau) = \frac{|\mathcal{M}|^4}{4}\Big\{&\frac{\gamma}{\Delta\Gamma}\big(e^{-2\gamma_L\tau}-e^{-2\gamma_S\tau}\big)\\
&+e^{-2\Gamma\tau}\Big[1-e^{-2(a+\alpha-\gamma)\tau}+\frac{a-\alpha}{\Delta m}\sin\big(2\Delta m\,\tau\big)+\frac{2b}{\Delta m}\Big(1-\cos(2\Delta m\,\tau)\Big)\pm\frac{16c\Delta\Gamma}{|\Delta\Gamma_+|^2}\Big]\end{aligned}$$

$$
\mp 8\frac{e^{-\Gamma\tau}}{|\Delta\Gamma_+|^2}\cos\big(\Delta m\,\tau\big)\Big[c\Delta\Gamma\big(e^{-\gamma_S\tau}+e^{-\gamma_L\tau}\big)+2\beta\Delta m\big(e^{-\gamma_S\tau}-e^{-\gamma_L\tau}\big)\Big]
$$

$$
\mp 8\frac{e^{-\Gamma\tau}}{|\Delta\Gamma_+|^2}\sin\big(\Delta m\,\tau\big)\Big[\beta\Delta\Gamma\big(e^{-\gamma_S\tau}+e^{-\gamma_L\tau}\big)-2c\Delta m\big(e^{-\gamma_S\tau}-e^{-\gamma_L\tau}\big)\Big]\Bigg\} \tag{4.14}
$$

$$
\mathcal{G}(\ell^\pm,\tau;\ell^\mp,\tau)=\frac{|\mathcal{M}|^4}{4}\Bigg\{\frac{\gamma}{\Delta\Gamma}\big(e^{-2\gamma_L\tau}-e^{-2\gamma_S\tau}\big)
$$

$$
+e^{-2\Gamma\tau}\Bigg[1+e^{-2(a+\alpha-\gamma)\tau}-\frac{a-\alpha}{\Delta m}\sin\big(2\Delta m\,\tau\big)-\frac{2b}{\Delta m}\Big(1-\cos\big(2\Delta m\,\tau\big)\Big)\Bigg]\Bigg\} \tag{4.15}
$$

In particular, the small time behaviour of these probabilities allow to obtain estimates on the non-standard parameter $a>0$:

$$
\frac{\mathcal{G}(\ell^\pm,\tau;\ell^\pm,\tau)}{\mathcal{G}(\ell^\pm,\tau;\ell^\mp,\tau)}\sim 2\ a\ \tau\ . \tag{4.16}
$$

All the correlation probabilities $\mathcal{G}(f_1,\tau_1;f_2,\tau_2)$ discussed so far can be measured at a ϕ-factory, and bounds on the various non-standard parameters a, b, c, α, β, γ can be obtained, at least in principle, by fitting the different time-behaviours. However, much of the analysis of ϕ-decay experiments has been carried out using integrated distributions at fixed time interval $\tau=\tau_1-\tau_2$.[21-24, 28] In the generic case, these single-time probabilities are defined as:

$$
\mathcal{F}(f_1,f_2;\tau)=\int_0^\infty dt\,\mathcal{G}(f_1,t+\tau;f_2,t)\ , \tag{4.17}
$$

where τ is taken to be positive. For negative τ, one defines:

$$
\mathcal{F}(f_1,f_2;-|\tau|)=\int_0^\infty dt\,\mathcal{G}(f_1,t-|\tau|;f_2,t)\ \theta(t-|\tau|)\ ; \tag{4.18}
$$

the presence of the step-function is necessary since the evolution is of semigroup type, with forward in time propagation, starting at the origin (we can not propagate a kaon before it is created in a ϕ-decay). In this case, one easily finds: $\mathcal{F}(f_1,f_2;-|\tau|)=\mathcal{F}(f_2,f_1;|\tau|)$. In the following, we shall always assume: $\tau\geq 0$.

With the integrated probabilities $\mathcal{F}$ one can form asymmetries that are sensitive to various parameters in the theory. For example, the following asymmetry involving two-pion decays:[28]

$$
\mathcal{A}_{\epsilon'}(\tau)=\frac{\mathcal{F}(\pi^+\pi^-,2\pi^0;\tau)-\mathcal{F}(2\pi^0,\pi^+\pi^-;\tau)}{\mathcal{F}(\pi^+\pi^-,2\pi^0;\tau)+\mathcal{F}(2\pi^0,\pi^+\pi^-;\tau)}\ , \tag{4.19}
$$

is particularly useful in the determination of the ratio ϵ'/ϵ, where ϵ and ϵ' are the phenomenological constants that parametrize the K_L, K_S two-pion decay amplitude ratios (see (4.7b) and the analogous expression in the case of neutral pions):

$$
\eta_{+-}=\epsilon+\epsilon'\ ,\qquad\qquad \eta_{00}=\epsilon-2\,\epsilon'\ . \tag{4.20}
$$

Using the relations in Appendix B, one can show that, to first order in ϵ'/ϵ:

$$\mathcal{A}_{\epsilon'}(\tau) = 3\,\mathcal{R}e\Big(\frac{\epsilon'}{\epsilon}\Big)\ \frac{N_1(\tau)}{D(\tau)} - 3\,\mathcal{I}m\Big(\frac{\epsilon'}{\epsilon}\Big)\ \frac{N_2(\tau)}{D(\tau)}\ . \tag{4.21}$$

The τ-dependent coefficients N_1, N_2 and D are functions of the non-standard parameters c, β, γ and of η_{+-}; their explicit expressions can be found in Appendix C.

From the experimental determination of the τ-dependence of $\mathcal{A}_{\epsilon'}(\tau)$ one can extract, at least in principle, the real and imaginary part of ϵ'/ϵ; however, notice that this is much more problematic than in the standard quantum mechanical case, in which N_1, N_2 and D have very simple τ-dependences. This has already been observed in [5] for the case of a simply-positive kaon dynamics, and remains true in the large time limit ($\tau \gg 1/\gamma_S$); in this limit, (4.21) becomes:

$$\begin{aligned}\mathcal{A}_{\epsilon'} \sim 3\,\mathcal{R}e\Big(\frac{\epsilon'}{\epsilon}\Big)\ \bigg\{1 - \frac{1}{|\eta_{+-}|^2}\bigg[\frac{\gamma}{\Delta\Gamma} - 8\bigg|\frac{C}{\Delta\Gamma_+}\bigg|^2 - 4\,\mathcal{R}e\bigg(\frac{\epsilon_L C}{\Delta\Gamma}\bigg)\bigg]&\\ - 4\,\mathcal{R}e\bigg(\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\bigg)\bigg\} - 12\,\mathcal{I}m\Big(\frac{\epsilon'}{\epsilon}\Big)\ \mathcal{I}m\bigg(\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\bigg)&\ .\end{aligned} \tag{4.22}$$

Similar results hold in the case of asymmetries involving semileptonic decays. The following three cases are of particular interest:[5, 28]

$$\mathcal{A}_L(\tau) = \frac{\mathcal{F}(\ell^+,\ell^+;\tau) - \mathcal{F}(\ell^-,\ell^+;\tau)}{\mathcal{F}(\ell^+,\ell^+;\tau) + \mathcal{F}(\ell^-,\ell^-;\tau)}\ , \tag{4.23a}$$

$$\mathcal{A}_T(\tau) = \frac{\mathcal{F}(\ell^+,\ell^+;\tau) - \mathcal{F}(\ell^-,\ell^-;\tau)}{\mathcal{F}(\ell^+,\ell^+;\tau) + \mathcal{F}(\ell^-,\ell^-;\tau)}\ , \tag{4.23b}$$

$$\mathcal{A}_{CPT}(\tau) = \frac{\mathcal{F}(\ell^+,\ell^-;\tau) - \mathcal{F}(\ell^-,\ell^+;\tau)}{\mathcal{F}(\ell^+,\ell^+;\tau) + \mathcal{F}(\ell^-,\ell^-;\tau)}\ . \tag{4.23c}$$

The first asymmetry is connected with the so called CP-violating charge asymmetry (see [28]), while $\mathcal{A}_T$ and $\mathcal{A}_{CPT}$ signal direct violation of time-reversal and CPT invariance. For simplicity, we again collect here only the large-time limit ($\tau \gg 1/\gamma_S$) of their expressions:

$$\mathcal{A}_L = 2\,\mathcal{R}e\bigg(\epsilon_L + \frac{2C^*}{\Delta\Gamma_-}\bigg) - \mathcal{R}e\big(x - z + 2\,y\big)\ , \tag{4.24a}$$

$$\begin{aligned}\mathcal{A}_T = 2\,\mathcal{R}e\bigg(\epsilon_L + \frac{2C^*}{\Delta\Gamma_-}\bigg) + 2\,\mathcal{R}e\bigg(\epsilon_S + \frac{2C}{\Delta\Gamma_-}\bigg)&\\ - 4\,\mathcal{R}e(y) - 16\,\mathcal{R}e\bigg(\frac{\Gamma\,C}{\Delta\Gamma_-(\Gamma_+ + \gamma_L)}\bigg)&\ ,\end{aligned} \tag{4.24b}$$

$$\begin{aligned}\mathcal{A}_{CPT} = 2\,\mathcal{R}e\bigg(\epsilon_L + \frac{2C^*}{\Delta\Gamma_-}\bigg) - 2\,\mathcal{R}e\bigg(\epsilon_S + \frac{2C}{\Delta\Gamma_-}\bigg)&\\ - 2\,\mathcal{R}e(x - z) + 16\,\mathcal{R}e\bigg(\frac{\Gamma\,C}{\Delta\Gamma_-(\Gamma_+ + \gamma_L)}\bigg)&\ .\end{aligned} \tag{4.24c}$$

Notice that the asymptotic value for $\mathcal{A}_L$ coincides with the charge asymmetry δ_L for the K_L state (see [19]).

Of great interest are also mixed asymmetries, involving both pion and semileptonic decays. These are particularly useful for studying CP-violating effects in $K_S \to 3\pi$ decays, which are hard to detect directly, due to the smallness of their branching ratios. Indeed, as suggested in [29, 28], one can alternatively study the following asymmetry, integrated over the whole Dalitz plot:

$$\mathcal{A}_{3\pi}(\tau) = \frac{\mathcal{F}(3\pi, \ell^+; \tau) - \mathcal{F}(3\pi, \ell^-; \tau)}{\mathcal{F}(3\pi, \ell^+; \tau) + \mathcal{F}(3\pi, \ell^-; \tau)}\ . \tag{4.25}$$

In the case of the $3\pi^0$ decay, one explicitly obtains:

$$\mathcal{A}_{3\pi^0}(\tau) = \frac{N_{3\pi^0}(\tau)}{D_{3\pi^0}(\tau)}\ , \tag{4.26}$$

where

$$\begin{aligned} N_{3\pi^0}(\tau) = {}& 2\,\mathcal{R}e\bigg(\epsilon_S + \frac{2C}{\Delta\Gamma_-} - \frac{8\Gamma\, C}{\Delta\Gamma_-(\Gamma_+ + \gamma_L)}\bigg) + \mathcal{R}e\big(x - z - 2\,y\big) \\ & - e^{-\Delta\Gamma\tau}\bigg(\frac{\gamma}{\Delta\Gamma}\bigg)\bigg[2\,\mathcal{R}e\bigg(\epsilon_L + \frac{2C}{\Delta\Gamma_+} - \frac{8\Gamma\, C}{\Delta\Gamma_+(\Gamma_+ + \gamma_S)}\bigg) - \mathcal{R}e\big(x - z + 2\,y\big)\bigg] \\ & - 2\,e^{-\Delta\Gamma\tau/2}\,\mathcal{R}e\Big(\eta_{000}\,e^{i\Delta m\tau}\Big)\ , \end{aligned} \tag{4.27a}$$

$$D_{3\pi^0}(\tau) = 1 + e^{-\Delta\Gamma\tau}\bigg[R^S_{000} + \frac{\gamma}{\Delta\Gamma}\bigg(\mathcal{R}e\big(x + z\big) + \frac{\gamma}{\Delta\Gamma}\,\frac{\gamma_L}{\gamma_S}\bigg)\bigg]\ , \tag{4.27b}$$

and R^S_{000} and η_{000} are obtained from (4.9a) and (4.9b) by replacing the parameter Y_{+-0} with Y_{000}. A similar expression holds for the $\pi^+\pi^-\pi^0$ asymmetry, provided one performs an even integration over the pion Dalitz plot. These asymmetries should be accessible to the experiment at a ϕ-factory.

5. DISCUSSION

As discussed in the previous section, the dynamics of the neutral kaon system based on the non-standard quantum evolution equation (2.3) can be probed at a ϕ-factory by measuring the time-evolution of various observables. Although the most striking differences with respect to the predictions of ordinary quantum mechanics arise at equal proper times, all the double probabilities computed in Section 4 have a non-trivial dependence on the parameters a, b, c, α, β and γ, with a corresponding characteristic time-behaviour. Therefore, by fitting the analytic expressions of these observables with the experimental data, it will be possible to obtain estimates for these parameters and, as a consequence, check the inequalities (2.11) they have to satisfy. Indeed, on phenomenological grounds, the values of the non-standard parameters may be expected to be of the order $m_k^2/m_P \sim 10^{-19}$GeV, with m_k the kaon mass and m_P the Planck mass, and therefore within the reach of an high luminosity ϕ-factory.[5] It should be stressed that having bounds on the values of a, b, c, α, β and γ is important also for the measure of the other parameters of the model, *e.g.* the ratio ϵ'/ϵ. In fact, the determination of the real and the imaginary part of this ratio via (4.21) and (4.22) is meaningful provided estimates on the non-standard parameters have been independently given.

A complete analysis on the sensitivities in the measure of a, b, c, α, β and γ that can be reached in an actual experimental setup at a ϕ-factory requires careful estimates of various effects, like the finite size of the detectors, and are certainly beyond the scope of the present paper. Nevertheless, we would like to comment on a possible background effect that could in principle interfere with the determination of a, b, c, α, β and γ.

Although not yet observed, the radiative decay $\phi \to \gamma\, K^0\overline{K^0}$ could constitute a non-vanishing source of background noise in a ϕ-factory experiment.[28] Due to the presence of the photon in the final state, the system of the two correlated neutral kaons are no longer initially described by (3.1), but rather by the symmetric state:

$$|\psi_S\rangle = \frac{1}{\sqrt{2}}\Big(|K_1,-p\rangle \otimes |K_2,p\rangle + |K_2,-p\rangle \otimes |K_1,p\rangle\Big)\ , \tag{5.1}$$

or equivalently by the density matrix:

$$\rho_S = \frac{1}{2}\Big[P_1 \otimes P_1\ +\ P_2 \otimes P_2\ -\ P_3 \otimes P_3\ -\ P_4 \otimes P_4\Big]\ . \tag{5.2}$$

Once created in a ϕ-decay, the state ρ_S will also evolve according with the completely positive dynamics regulated by the equation (2.3) and contribute to the double probability distributions $\mathcal{G}(f_1,\tau_1;f_2,\tau_2)$ considered in the previous section. Although these additional contributions are suppressed by the branching ratio

$$r = \frac{\mathcal{A}(\phi \to \gamma K^0\overline{K^0})}{\mathcal{A}(\phi \to K^0\overline{K^0})}\ , \tag{5.3}$$

they could give rise to non-negligible effects.

In order to check this possibility, we have computed the ρ_S-contributions to the three probabilities distributions $\mathcal{G}$ involving equal final states ($\pi^+\pi^-$, $\pi^+\pi^-\pi^0$ and $\pi^-\ell^+\nu$); the explicit expressions are collected in Appendix D. One observes that the dependence on the parameters of the model, as well as the time-dependence of these contributions differ from the ones described in the previous section. Therefore, at least in principle, it is possible to isolate the background ρ_S-contribution from the interesting ρ_A-terms.

However, if the branching ratio r is of order $10^{-7} - 10^{-8}$, as some theoretical investigations seem to suggest,[30-32] there are instances where the symmetric background contribution can not be neglected. This is the case for the asymptotic long-time behaviour of the equal-time probability $\mathcal{G}(\pi^+\pi^-\pi^0,\tau\,;\pi^+\pi^-\pi^0,\tau)$; using the results of Appendix D, in the presence of the ρ_S-background the asymptotic behaviour in (4.10) is modified as follows:

$$\mathcal{G}(\pi^+\pi^-\pi^0,\tau\,;\pi^+\pi^-\pi^0,\tau) \sim \left(\frac{\gamma}{\Delta\Gamma} + \frac{r}{2}\right)\ e^{-2\gamma_L\tau}\ , \tag{5.4}$$

and one needs to study the full double-time correlation $\mathcal{G}(\pi^+\pi^-\pi^0, \tau_1\,; \pi^+\pi^-\pi^0, \tau_2)$ in order to get an estimate for the parameter γ. On the other hand, the correction to the ratio of semileptonic probabilities in (4.16) is proportional to r times terms linear in the non-standard parameters, and therefore can be totally ignored. One should also notice that the effect of the background $\phi \to \gamma K^0 \overline{K^0}$ can be greatly reduced by a careful choice of the geometry of the measuring detector.[33] In conclusion, a high-luminosity ϕ-factory can put stringent limits on the values of the parameters a, b, c, α, β and γ, while keeping under control possible background effects.

As a final remark, we would like to point out another unique feature of the ϕ-interferometry physics. All the considerations of the previous sections are based on the assumption of a completely positive dynamics for the neutral kaon system. Had we used a simply positive time-evolution instead, inconsistencies in the formalism would have emerged in the study of correlated kaons. One of the properties of the density matrix $\rho_A(\tau) \equiv \rho_A(\tau, \tau)$ in (3.4) is that its mean value on any vector is positive for all times. Without this basic requirement, the standard probability interpretation of $\rho_A(\tau)$ as a state of the correlated kaons would be meaningless.

In the case of the vector:

$$|u\rangle = \frac{1}{\sqrt{2}} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right] \ , \tag{5.5}$$

the completely positive dynamics of equation (2.3) gives, to first order in the small parameters:

$$\begin{aligned} \mathcal{U}(\tau) \equiv \langle u|\rho_A(\tau)|u\rangle = \Bigg\{ & \frac{\gamma}{2\Delta\Gamma} \left(e^{-2\gamma_L \tau} \ - \ e^{-2\gamma_S \tau} \right) \\ & - e^{-2\Gamma\tau} \ \left[\frac{\alpha - a}{2\Delta m} \ \sin\left(2\Delta m\, \tau\right) \ + \ \frac{b}{\Delta m} \Big(\cos\left(2\Delta m\, \tau\right) - 1 \Big) \right] \Bigg\} \ . \end{aligned} \tag{5.6}$$

This result is indeed positive for all times, and vanish only at $\tau = 0$, due to the antisymmetry of ρ_A in (3.3).

This conclusion is not true in general if the time evolution for ρ_A is only positive and not completely positive. For instance, if one chooses to work with the simply positive dynamics studied in [1-4, 5], that can be obtained from the one in (2.3), (2.10) by setting $a = b = c = 0$, then the mean value $\mathcal{U}(\tau)$ would be negative for small enough times, signaling the presence of unphysical negative probabilities (see also [19]). A ϕ-factory, being a high performance quantum interferometer, can measure, at least in principle, observables like (5.6) and therefore further clarify the request of complete positivity for the dynamics of neutral kaons.

APPENDIX A

In order to calculate the explicit expressions of the observables $\mathcal{G}(f_1,\tau_1;f_2,\tau_2)$ in (3.5) we need to determine the time evolution of the single kaon operators P_i, $i=1,2,3,4$, in (3.2). Using (2.13), with (3.2) as initial conditions, one finds

$$P_1(\tau)=\begin{pmatrix} M_{11}^{(0)}(\tau)+M_{11}^{(2)}(\tau) & M_{31}^{(1)}(\tau)+M_{31}^{(2)}(\tau) \\ M_{41}^{(1)}(\tau)+M_{41}^{(2)}(\tau) & M_{21}^{(1)}(\tau)+M_{21}^{(2)}(\tau) \end{pmatrix} \tag{A.1a}$$

$$P_2(\tau)=\begin{pmatrix} M_{12}^{(1)}(\tau)+M_{12}^{(2)}(\tau) & M_{32}^{(1)}(\tau)+M_{32}^{(2)}(\tau) \\ M_{42}^{(1)}(\tau)+M_{42}^{(2)}(\tau) & M_{22}^{(0)}(\tau)+M_{22}^{(2)}(\tau) \end{pmatrix} \tag{A.1b}$$

$$P_3(\tau)=\begin{pmatrix} M_{13}^{(1)}(\tau)+M_{13}^{(2)}(\tau) & M_{33}^{(0)}(\tau)+M_{33}^{(2)}(\tau) \\ M_{43}^{(1)}(\tau)+M_{43}^{(2)}(\tau) & M_{23}^{(1)}(\tau)+M_{23}^{(2)}(\tau) \end{pmatrix} \tag{A.1c}$$

$$P_4(\tau)=\begin{pmatrix} M_{14}^{(1)}(\tau)+M_{14}^{(2)}(\tau) & M_{34}^{(1)}(\tau)+M_{34}^{(2)}(\tau) \\ M_{44}^{(0)}(\tau)+M_{44}^{(2)}(\tau) & M_{24}^{(1)}(\tau)+M_{24}^{(2)}(\tau) \end{pmatrix}\ . \tag{A.1d}$$

With these expressions, one can now calculate the "elementary" double probabilities in (3.8):

$$\begin{aligned}\mathcal{P}_{ij}(\tau_1,\tau_2)=\mathrm{Tr}\big\{P_i\,P_1(\tau_1)\big\}\ \mathrm{Tr}\big\{P_j\,P_2(\tau_2)\big\}\ &+\ \mathrm{Tr}\big\{P_i\,P_2(\tau_1)\big\}\ \mathrm{Tr}\big\{P_j\,P_1(\tau_2)\big\}\\ -\ \mathrm{Tr}\big\{P_i\,P_3(\tau_1)\big\}\ \mathrm{Tr}\big\{P_j\,P_4(\tau_2)\big\}\ &-\ \mathrm{Tr}\big\{P_i\,P_4(\tau_1)\big\}\ \mathrm{Tr}\big\{P_j\,P_3(\tau_2)\big\}\ .\end{aligned} \tag{A.2}$$

They satisfy the following symmetry property:

$$\mathcal{P}_{ij}(\tau_1,\tau_2)=\mathcal{P}_{ji}(\tau_2,\tau_1)\ ; \tag{A.3}$$

further, since the single kaon dynamics preserves hermiticity,

$$\mathcal{P}_{ij}(\tau_1,\tau_2)=\mathcal{P}_{ij}(\tau_1,\tau_2)^*\ ,\quad i,j=1,2\ , \tag{A.4a}$$

$$\mathcal{P}_{ij}(\tau_1,\tau_2)^*=\mathcal{P}_{ji}(\tau_1,\tau_2)\ ,\quad i,j=3,4\ , \tag{A.4b}$$

$$\mathcal{P}_{i3}(\tau_1,\tau_2)^*=\mathcal{P}_{i4}(\tau_1,\tau_2)\ ,\quad i=1,2\ . \tag{A.4c}$$

It then follows:

$$\begin{aligned}
\mathcal{P}_{11}(\tau_1,\tau_2) &= M_{11}^{(0)}(\tau_1)\ M_{12}^{(1)}(\tau_2) + M_{12}^{(1)}(\tau_1)\ M_{11}^{(0)}(\tau_2) \\
&\quad + M_{11}^{(0)}(\tau_1)\ M_{12}^{(2)}(\tau_2) + M_{12}^{(2)}(\tau_1)\ M_{11}^{(0)}(\tau_2) \qquad (A.5a)\\
&\quad - 2\,\mathcal{R}e\Big(M_{13}^{(1)}(\tau_1)\ M_{14}^{(1)}(\tau_2)\Big)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{12}(\tau_1,\tau_2) &= M_{11}^{(0)}(\tau_1)\ M_{22}^{(0)}(\tau_2) \\
&\quad + M_{11}^{(0)}(\tau_1)\ M_{22}^{(2)}(\tau_2) + M_{11}^{(2)}(\tau_1)\ M_{22}^{(0)}(\tau_2) \qquad (A.5b)\\
&\quad + M_{12}^{(1)}(\tau_1)\ M_{21}^{(1)}(\tau_2) - 2\,\mathcal{R}e\ \Big(M_{13}^{(1)}(\tau_1)\ M_{24}^{(1)}(\tau_2)\Big)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{13}(\tau_1,\tau_2) &= M_{11}^{(0)}(\tau_1)\ M_{42}^{(1)}(\tau_2) - M_{13}^{(1)}(\tau_1)\ M_{44}^{(0)}(\tau_2) \\
&\quad + M_{11}^{(0)}(\tau_1)\ M_{42}^{(2)}(\tau_2) - M_{13}^{(2)}(\tau_1)\ M_{44}^{(0)}(\tau_2) \qquad (A.5c)\\
&\quad + M_{12}^{(1)}(\tau_1)\ M_{41}^{(1)}(\tau_2) - M_{14}^{(1)}(\tau_1)\ M_{43}^{(1)}(\tau_2)
\end{aligned}$$

$$\mathcal{P}_{14}(\tau_1,\tau_2) = \mathcal{P}_{13}(\tau_1,\tau_2)^* \qquad (A.5d)$$

$$\mathcal{P}_{21}(\tau_1,\tau_2) = \mathcal{P}_{12}(\tau_2,\tau_1) \qquad (A.6a)$$

$$\begin{aligned}
\mathcal{P}_{22}(\tau_1,\tau_2) &= M_{21}^{(1)}(\tau_1)\ M_{22}^{(0)}(\tau_2) + M_{22}^{(0)}(\tau_1)\ M_{21}^{(1)}(\tau_2) \\
&\quad + M_{21}^{(2)}(\tau_1)\ M_{22}^{(0)}(\tau_2) + M_{22}^{(0)}(\tau_1)\ M_{21}^{(2)}(\tau_2) \qquad (A.6b)\\
&\quad - 2\,\mathcal{R}e\ \Big(M_{23}^{(1)}(\tau_1)\ M_{24}^{(1)}(\tau_2)\Big)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{23}(\tau_1,\tau_2) &= M_{22}^{(0)}(\tau_1)\ M_{41}^{(1)}(\tau_2) - M_{23}^{(1)}(\tau_1)\ M_{44}^{(0)}(\tau_2) \\
&\quad + M_{22}^{(0)}(\tau_1)\ M_{41}^{(2)}(\tau_2) - M_{23}^{(2)}(\tau_1)\ M_{44}^{(0)}(\tau_2) \qquad (A.6c)\\
&\quad + M_{21}^{(1)}(\tau_1)\ M_{42}^{(1)}(\tau_2) - M_{24}^{(1)}(\tau_1)\ M_{43}^{(1)}(\tau_2)
\end{aligned}$$

$$\mathcal{P}_{24}(\tau_1,\tau_2) = \mathcal{P}_{23}(\tau_1,\tau_2)^* \qquad (A.6d)$$

$$\mathcal{P}_{31}(\tau_1,\tau_2) = \mathcal{P}_{13}(\tau_2,\tau_1) \qquad (A.7a)$$

$$\mathcal{P}_{32}(\tau_1,\tau_2) = \mathcal{P}_{23}(\tau_2,\tau_1) \qquad (A.7b)$$

$$\begin{aligned}
\mathcal{P}_{33}(\tau_1,\tau_2) &= -M_{43}^{(1)}(\tau_1)\ M_{44}^{(0)}(\tau_2) - M_{44}^{(0)}(\tau_1)\ M_{43}^{(1)}(\tau_2) \\
&\quad - M_{43}^{(2)}(\tau_1)\ M_{44}^{(0)}(\tau_2) - M_{44}^{(0)}(\tau_1)\ M_{43}^{(2)}(\tau_2) \qquad (A.7c)\\
&\quad + M_{41}^{(1)}(\tau_1)\ M_{42}^{(1)}(\tau_2) + M_{42}^{(1)}(\tau_1)\ M_{41}^{(1)}(\tau_2)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{34}(\tau_1,\tau_2) &= -M_{44}^{(0)}(\tau_1)\ M_{33}^{(0)}(\tau_2) \\
&\quad - M_{44}^{(0)}(\tau_1)\ M_{33}^{(2)}(\tau_2) - M_{44}^{(2)}(\tau_1)\ M_{33}^{(0)}(\tau_2) \qquad (A.7d)\\
&\quad + M_{41}^{(1)}(\tau_1)\ M_{32}^{(1)}(\tau_2) + M_{42}^{(1)}(\tau_1)\ M_{31}^{(1)}(\tau_2) - M_{43}^{(1)}(\tau_1)\ M_{34}^{(1)}(\tau_2)
\end{aligned}$$

$$\mathcal{P}_{41}(\tau_1,\tau_2) = \mathcal{P}_{31}(\tau_1,\tau_2)^* \qquad (A.8a)$$

$$\mathcal{P}_{42}(\tau_1,\tau_2) = \mathcal{P}_{32}(\tau_1,\tau_2)^* \qquad (A.8b)$$

$$\mathcal{P}_{43}(\tau_1,\tau_2)=\mathcal{P}_{34}(\tau_2,\tau_1) \qquad (A.8c)$$
$$\mathcal{P}_{44}(\tau_1,\tau_2)=\mathcal{P}_{33}(\tau_1,\tau_2)^* \ . \qquad (A.8d)$$

Using the expressions for $M_{ij}^{(0)}$ and $M_{ij}^{(1)}$ given in Section 2, together with the ones collected in the Appendix of [17] for $M_{ij}^{(2)}$, one arrives at the formulas listed below. As for the entries of $M_{ij}^{(0)}$, $M_{ij}^{(1)}$ and $M_{ij}^{(2)}$, in order to preserve the exponential form of the time evolution, linear and quadratic corrections in τ_1 and τ_2 have been reabsorbed in the exponents.

$$\begin{aligned}
\mathcal{P}_{11}(\tau_1,\tau_2) &= e^{-\gamma_S(\tau_1+\tau_2)}\bigg[-2\frac{\gamma}{\Delta\Gamma}+\frac{8}{\Delta\Gamma}\mathcal{R}e(C\epsilon_L)-16\left|\frac{C}{\Delta\Gamma_+}\right|^2\bigg]\\
&+\left(e^{-\gamma_S\tau_1-\gamma_L\tau_2}+e^{-\gamma_L\tau_1-\gamma_S\tau_2}\right)\bigg[\frac{\gamma}{\Delta\Gamma}+|e_L(+,-)|^2-\frac{4}{\Delta\Gamma}\mathcal{R}e(C\epsilon_L)-8\left|\frac{C}{\Delta\Gamma_+}\right|^2\bigg]\\
&-\mathcal{R}e\bigg[\left(e^{-\gamma_S\tau_1-\Gamma_+\tau_2}+e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\right)\ \frac{8C}{\Delta\Gamma_+}e_L(-,-)\bigg]\\
&-2\,\mathcal{R}e\bigg[e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}\ |e_L(-,-)|^2\bigg] \qquad (A.9a)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{12}(\tau_1,\tau_2) &= \left(2\,e^{-\gamma_S(\tau_1+\tau_2)}+e^{-\gamma_L(\tau_1+\tau_2)}-e^{-\gamma_L\tau_1-\gamma_S\tau_2}\right)\left(\frac{\gamma}{\Delta\Gamma}\right)^2\\
&+e^{-\gamma_S\tau_1-\gamma_L\tau_2}\bigg[1-3\left(\frac{\gamma}{\Delta\Gamma}\right)^2+2\,\mathcal{R}e\left(\epsilon_S\epsilon_L+2\frac{C^*\epsilon_S+C\epsilon_L}{\Delta\Gamma_-}\right)-24\,\mathcal{R}e\left(\frac{|C|^2}{(\Delta\Gamma_+)^2}\right)\bigg]\\
&-\mathcal{R}e\bigg[e^{-\gamma_S\tau_1-\Gamma_-\tau_2}\ \frac{8C^*}{\Delta\Gamma_-}f_S(-,-)\bigg]-\mathcal{R}e\bigg[e^{-\Gamma_-\tau_1-\gamma_L\tau_2}\ \frac{8C^*}{\Delta\Gamma_+}e_L^*(-,-)\Big)\bigg]\\
&-2\,\mathcal{R}e\bigg[e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}\ e_L(-,-)f_S(-,-)\bigg] \qquad (A.9b)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{13}(\tau_1,\tau_2) &= \left(e^{-\gamma_L\tau_1-\gamma_S\tau_2}-2\,e^{-\gamma_S(\tau_1+\tau_2)}\right)\ \frac{\gamma}{\Delta\Gamma}\,f_S(+,-)\\
&+e^{-\gamma_S\tau_1-\gamma_L\tau_2}\bigg[\left(1-\frac{2(A-\gamma)}{\Delta\Gamma_+}\right)e_L^*(+,-)+\frac{2B^*}{\Delta\Gamma_+}e_L(+,-)-\frac{\gamma}{\Delta\Gamma}\frac{\Delta\Gamma_-}{\Delta\Gamma_+}f_S(+,-)\bigg]\\
&-e^{-\gamma_S\tau_1-(\Gamma_++A-\gamma)\tau_2}\bigg[4\frac{C}{\Delta\Gamma_+}-\frac{4(A-\gamma)}{\Delta\Gamma_+}\epsilon_L^*-\frac{4\gamma\epsilon_S}{\Delta\Gamma_+}-\frac{4\gamma C}{\Delta\Gamma\Delta\Gamma_-}+\frac{4B^*\epsilon_L}{\Delta\Gamma_+}+\frac{2iB^*C^*}{\Delta m\Delta\Gamma_-}\bigg]\\
&+e^{-\gamma_S\tau_1-\Gamma_-\tau_2}\ \frac{2iB^*C^*}{\Delta m\Delta\Gamma_-}-e^{-\gamma_L\tau_1-\Gamma_+\tau_2}\ \frac{4\gamma C}{\Delta\Gamma\Delta\Gamma_-}\\
&-e^{-\Gamma_-\tau_1-\Gamma_+\tau_2-(A-\gamma)(\tau_1+\tau_2)}\bigg[\left(1+\frac{2(A-\gamma)}{\Delta\Gamma_+}\right)e_L^*(-,-)+\frac{2\gamma}{\Delta\Gamma_+}f_S(-,-)\\
&-\frac{iB^*}{2\Delta m}\frac{\Delta\Gamma_-}{\Delta\Gamma_+}e_L(-,-)\bigg]+e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}\ \frac{iB^*}{2\Delta m}e_L(-,-)-e^{-\Gamma_+(\tau_1+\tau_2)}\ \frac{iB^*}{\Delta m}e_L(-,-)
\end{aligned}$$
$$(A.9c)$$

$$\mathcal{P}_{22}(\tau_1,\tau_2) = \left(e^{-\gamma_S\tau_1-\gamma_L\tau_2} + e^{-\gamma_L\tau_1-\gamma_S\tau_2}\right)\left[|f_S(+,-)|^2 - \frac{\gamma}{\Delta\Gamma} - \frac{4}{\Delta\Gamma}\mathcal{R}e(C^*\epsilon_S) - 8\left|\frac{C}{\Delta\Gamma_+}\right|^2\right]$$
$$+ e^{-\gamma_L(\tau_1+\tau_2)}\left[2\frac{\gamma}{\Delta\Gamma} + \frac{8}{\Delta\Gamma}\mathcal{R}e(C^*\epsilon_S) - 16\left|\frac{C}{\Delta\Gamma_+}\right|^2\right]$$
$$- \mathcal{R}e\left[\left(e^{-\gamma_L\tau_1-\Gamma_-\tau_2} + e^{-\Gamma_-\tau_1-\gamma_L\tau_2}\right)\ \frac{8C^*}{\Delta\Gamma_+}f_S(-,-)\right]$$
$$- 2\,\mathcal{R}e\left[e^{-\Gamma_-\tau_1-\Gamma_+\tau_2}\ |f_S(-,-)|^2\right] \qquad (A.9d)$$

$$\mathcal{P}_{23}(\tau_1,\tau_2) = \left(2\,e^{-\gamma_L(\tau_1+\tau_2)} - e^{-\gamma_S\tau_1-\gamma_L\tau_2}\right)\ \frac{\gamma}{\Delta\Gamma}\,e_L^*(+,-)$$
$$+ e^{-\gamma_L\tau_1-\gamma_S\tau_2}\left[\left(1+\frac{2(A-\gamma)}{\Delta\Gamma_-}\right)f_S(+,-) + \frac{\gamma}{\Delta\Gamma}\frac{\Delta\Gamma_+}{\Delta\Gamma_-}e_L^*(+,-) - \frac{2B^*}{\Delta\Gamma_-}f_S^*(+,-)\right]$$
$$- e^{-\gamma_L\tau_1-(\Gamma_++A-\gamma)\tau_2}\left[4\frac{C}{\Delta\Gamma_-} + \frac{4(A-\gamma)}{\Delta\Gamma_-}\epsilon_S + \frac{4\gamma\epsilon_L^*}{\Delta\Gamma_-} + \frac{4\gamma C}{\Delta\Gamma\Delta\Gamma_+} - \frac{4B^*\epsilon_S^*}{\Delta\Gamma_-} + \frac{2iB^*C^*}{\Delta m\Delta\Gamma_+}\right]$$
$$+ e^{-\gamma_S\tau_1-\Gamma_+\tau_2}\ \frac{4\gamma C}{\Delta\Gamma\Delta\Gamma_+} + e^{-\gamma_L\tau_1-\Gamma_-\tau_2}\ \frac{2iB^*C^*}{\Delta m\Delta\Gamma_+}$$
$$- e^{-\Gamma_-\tau_1-\Gamma_+\tau_2-(A-\gamma)(\tau_1+\tau_2)}\left[\left(1-\frac{2(A-\gamma)}{\Delta\Gamma_-}\right)f_S(-,-) - \frac{2\gamma}{\Delta\Gamma_-}e_L^*(-,-)\right.$$
$$\left.- \frac{iB^*}{2\Delta m}\frac{\Delta\Gamma_+}{\Delta\Gamma_-}f_S^*(-,-)\right] - e^{-\Gamma_+(\tau_1+\tau_2)}\frac{iB^*}{\Delta m}f_S^*(-,-) + e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}\frac{iB^*}{2\Delta m}f_S^*(-,-)$$
$$(A.9e)$$

$$\mathcal{P}_{33}(\tau_1,\tau_2) = \left(e^{-\gamma_S\tau_1-\gamma_L\tau_2} + e^{-\gamma_L\tau_1-\gamma_S\tau_2}\right)\ e_L^*(+,-)f_S(+,-)$$
$$- \left(e^{-\gamma_S\tau_1-\Gamma_+\tau_2} + e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\right)\ \frac{4C}{\Delta\Gamma_+}f_S(+,-)$$
$$- \left(e^{-\gamma_L\tau_1-\Gamma_+\tau_2} + e^{-\Gamma_+\tau_1-\gamma_L\tau_2}\right)\ \frac{4C}{\Delta\Gamma_-}e_L^*(+,-)$$
$$+ e^{-(\Gamma_++A-\gamma)(\tau_1+\tau_2)}\left[-\frac{iB^*}{\Delta m} - 2\frac{iC(\epsilon_S-\epsilon_L^*)}{\Delta m} + 16\frac{C^2}{|\Delta\Gamma_+|^2}\right]$$
$$+ \left(e^{-\Gamma_+\tau_1-\Gamma_-\tau_2} + e^{-\Gamma_-\tau_1-\Gamma_+\tau_2}\right)\ e^{-(A-\gamma)(\tau_1+\tau_2)}$$
$$\times\ \left[\frac{iB^*}{2\Delta m} - e_L^*(-,-)f_S(-,-) + i\frac{C(\epsilon_S-\epsilon_L^*)}{\Delta m} + 8\frac{C^2}{|\Delta\Gamma_+|^2}\right]$$
$$(A.9f)$$

$$
\begin{aligned}
\mathcal{P}_{34}(\tau_1,\tau_2) = \; & e^{-\gamma_S\tau_1-\gamma_L\tau_2}\; e_L(+,-) f_S(+,-) + e^{-\gamma_L\tau_1-\gamma_S\tau_2}\; e_L^*(+,-) f_S^*(+,-)\\
& - e^{-\gamma_S\tau_1-\Gamma_-\tau_2}\; \frac{4C^*}{\Delta\Gamma_-} f_S(+,-) - e^{-\gamma_L\tau_1-\Gamma_-\tau_2}\; \frac{4C^*}{\Delta\Gamma_+} e_L^*(+,-)\\
& - e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\; \frac{4C}{\Delta\Gamma_+} f_S^*(+,-) - e^{-\Gamma_+\tau_1-\gamma_L\tau_2}\; \frac{4C}{\Delta\Gamma_-} e_L(+,-)\\
& - e^{-\widetilde{\Gamma}_+\tau_1-\widetilde{\Gamma}_-\tau_2}\Big[1 + 3\frac{|B|^2}{4(\Delta m)^2} + 2\,\mathcal{R}e\Big(\epsilon_S\epsilon_L - 2\frac{\epsilon_S C^* + \epsilon_L C}{\Delta\Gamma_-}\Big) - 24\mathcal{R}e\Big(\frac{|C|^2}{(\Delta\Gamma_+)^2}\Big)\Big]\\
& + \Big(2\, e^{-\Gamma_+(\tau_1+\tau_2)} + 2\, e^{-\Gamma_-(\tau_1+\tau_2)} - e^{-\Gamma_-\tau_1-\Gamma_+\tau_2}\Big)\; \frac{|B|^2}{4(\Delta m)^2}\ . \qquad (A.9g)
\end{aligned}
$$

APPENDIX B

We collect here the explicit expressions for the double correlations $\mathcal{G}(f_1,\tau_1;f_2,\tau_2)$. For the observables having pions in the final states, as in Section 4, it is convenient to introduce the following quantities:

$$
\begin{aligned}
& R^L_{2\pi} = \Big|\epsilon_L + \frac{2C^*}{\Delta\Gamma_-} + Y_{2\pi}\Big|^2 + \frac{\gamma}{\Delta\Gamma} - 8\Big|\frac{C}{\Delta\Gamma_+}\Big|^2 - \frac{4}{\Delta\Gamma}\mathcal{R}e(C\epsilon_L)\\
& \eta_{2\pi} = \epsilon_L - \frac{2C^*}{\Delta\Gamma_-} + Y_{2\pi}\ , \qquad (B.1)\\
& R^S_{3\pi} = \Big|\epsilon_S + \frac{2C}{\Delta\Gamma_-} + Y_{3\pi}\Big|^2 - \frac{\gamma}{\Delta\Gamma} - 8\Big|\frac{C}{\Delta\Gamma_+}\Big|^2 - \frac{4}{\Delta\Gamma}\mathcal{R}e(C^*\epsilon_S)\\
& \eta_{3\pi} = \epsilon_S - \frac{2C}{\Delta\Gamma_-} + Y_{3\pi}\ , \qquad (B.2)
\end{aligned}
$$

where the indices 2π and 3π can assume the values $+-$, 00 and $+-0$, 000, respectively, according to the charge of the pions. Then, using the expressions derived in the previous Appendix, one explicitly obtains, up to second order in all small parameters:

$$
\begin{aligned}
\mathcal{G}(\pi^+\pi^-,\tau_1;\pi^+\pi^-,\tau_2) = \frac{|X_{+-}|^4}{2}\Big\{ & e^{-\gamma_S(\tau_1+\tau_2)}\Big[-2\frac{\gamma}{\Delta\Gamma} + \frac{8}{\Delta\Gamma}\mathcal{R}e\big(C\epsilon_L\big) - 16\Big|\frac{C}{\Delta\Gamma_+}\Big|^2\Big]\\
& + \Big(e^{-\gamma_S\tau_1-\gamma_L\tau_2} + e^{-\gamma_L\tau_1-\gamma_S\tau_2}\Big)\; R^L_{+-}\\
& - \mathcal{R}e\Big[\frac{8C}{\Delta\Gamma_+}\eta_{+-}\Big(e^{-\gamma_S\tau_1-\Gamma_+\tau_2} + e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\Big) + 2|\eta_{+-}|^2\; e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}\Big]\Big\} \qquad (B.3a)
\end{aligned}
$$

$$\mathcal{G}(\pi^+\pi^-,\tau_1;2\pi^0,\tau_2)=\frac{|X_{+-}|^2|X_{00}|^2}{2}\Bigg\{e^{-\gamma_S(\tau_1+\tau_2)}\Bigg[-2\frac{\gamma}{\Delta\Gamma}+\frac{8}{\Delta\Gamma}\mathcal{R}e\big(C\epsilon_L\big)-16\left|\frac{C}{\Delta\Gamma_+}\right|^2\Bigg]$$
$$+e^{-\gamma_S\tau_1-\gamma_L\tau_2}\ R^L_{00}+e^{-\gamma_L\tau_1-\gamma_S\tau_2}\ R^L_{+-}$$
$$-\mathcal{R}e\Bigg[\frac{8C}{\Delta\Gamma_+}\Big(\eta_{00}\ e^{-\gamma_S\tau_1-\Gamma_+\tau_2}+\eta_{+-}\ e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\Big)+2\,\eta^*_{+-}\,\eta_{00}\ e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}\Bigg]\Bigg\}\qquad(B.3b)$$

$$\mathcal{G}(\pi^+\pi^-\pi^0,\tau_1;\pi^+\pi^-\pi^0,\tau_2)=\frac{|X_{+-0}|^4}{2}\Bigg\{e^{-\gamma_L(\tau_1+\tau_2)}\Bigg[2\frac{\gamma}{\Delta\Gamma}+\frac{8}{\Delta\Gamma}\mathcal{R}e\big(C^*\epsilon_S\big)-16\left|\frac{C}{\Delta\Gamma_+}\right|^2\Bigg]$$
$$+\Big(e^{-\gamma_S\tau_1-\gamma_L\tau_2}+e^{-\gamma_L\tau_1-\gamma_S\tau_2}\Big)\ R^S_{+-0}$$
$$-\mathcal{R}e\Bigg[\frac{8C^*}{\Delta\Gamma_+}\eta_{+-0}\Big(e^{-\gamma_L\tau_1-\Gamma_-\tau_2}+e^{-\Gamma_-\tau_1-\gamma_L\tau_2}\Big)+2|\eta_{+-0}|^2\ e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}\Bigg]\Bigg\}\qquad(B.3c)$$

$$\mathcal{G}(\pi^+\pi^-\pi^0,\tau_1;3\pi^0,\tau_2)=\frac{|X_{+-0}|^2|X_{000}|^2}{2}\Bigg\{e^{-\gamma_L(\tau_1+\tau_2)}\Bigg[2\frac{\gamma}{\Delta\Gamma}+\frac{8}{\Delta\Gamma}\mathcal{R}e\big(C^*\epsilon_S\big)-16\left|\frac{C}{\Delta\Gamma_+}\right|^2\Bigg]$$
$$+e^{-\gamma_S\tau_1-\gamma_L\tau_2}\ R^S_{+-0}+e^{-\gamma_L\tau_1-\gamma_S\tau_2}\ R^S_{000}$$
$$-\mathcal{R}e\Bigg[\frac{8C^*}{\Delta\Gamma_+}\Big(\eta_{000}\,e^{-\gamma_L\tau_1-\Gamma_-\tau_2}+\eta_{+-0}\,e^{-\Gamma_-\tau_1-\gamma_L\tau_2}\Big)+2\,\eta_{000}\,\eta^*_{+-0}\ e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}\Bigg]\Bigg\}$$
$$(B.3d)$$

The semileptonic correlations (4.13) need first order contributions only:

$$\mathcal{G}(\ell^+,\tau_1;\ell^-,\tau_2)=\frac{|\mathcal{M}|^4}{8}\Bigg\{\frac{2\gamma}{\Delta\Gamma}\Big(e^{-\gamma_L(\tau_1+\tau_2)}-e^{-\gamma_S(\tau_1+\tau_2)}\Big)+e^{-\gamma_S\tau_1-\gamma_L\tau_2}+e^{-\gamma_L\tau_1-\gamma_S\tau_2}$$
$$+2\,\mathcal{R}e\Bigg[\epsilon_S-\epsilon_L+x-z+\frac{4i\beta}{\Delta\Gamma_-}\Bigg]\Big(e^{-\gamma_S\tau_1-\gamma_L\tau_2}-e^{-\gamma_L\tau_1-\gamma_S\tau_2}\Big)$$
$$+\mathcal{R}e\Bigg[\frac{8C}{\Delta\Gamma_+}\Big(e^{-\gamma_S\tau_1-\Gamma_+\tau_2}-e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\Big)\Bigg]+\mathcal{R}e\Bigg[\frac{8C}{\Delta\Gamma_-}\Big(e^{-\gamma_L\tau_1-\Gamma_+\tau_2}-e^{-\Gamma_+\tau_1-\gamma_L\tau_2}\Big)\Bigg]$$
$$+e^{-\Gamma(\tau_1+\tau_2)}\Bigg[\frac{4b}{\Delta m}\cos\big(\Delta m(\tau_1+\tau_2)\big)-\Bigg(\frac{4b}{\Delta m}-2e^{-(A-\gamma)(\tau_1+\tau_2)}\Bigg)\cos\big(\Delta m(\tau_1-\tau_2)\big)$$
$$+\frac{2(\alpha-a)}{\Delta m}\sin\big(\Delta m(\tau_1+\tau_2)\big)+4\,\mathcal{I}m\Bigg(\epsilon_L-\epsilon_S+z-x+\frac{4i\beta}{\Delta\Gamma_-}\Bigg)\ \sin\big(\Delta m(\tau_1-\tau_2)\big)\Bigg]\Bigg\}$$
$$(B.4a)$$

$$\mathcal{G}(\ell^+,\tau_1;\ell^+,\tau_2)=\frac{|\mathcal{M}|^4}{8}\Bigg\{\frac{2\gamma}{\Delta\Gamma}\Big(e^{-\gamma_L(\tau_1+\tau_2)}-e^{-\gamma_S(\tau_1+\tau_2)}\Big)$$
$$+\left[1+2\,\mathcal{R}e\Big(\epsilon_S+\epsilon_L-2y+\frac{4c}{\Delta\Gamma_-}\Big)\right]\Big(e^{-\gamma_S\tau_1-\gamma_L\tau_2}+e^{-\gamma_L\tau_1-\gamma_S\tau_2}\Big)$$
$$-\mathcal{R}e\left[\frac{8C}{\Delta\Gamma_+}\Big(e^{-\gamma_S\tau_1-\Gamma_+\tau_2}+e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\Big)\right]-\mathcal{R}e\left[\frac{8C}{\Delta\Gamma_-}\Big(e^{-\gamma_L\tau_1-\Gamma_+\tau_2}+e^{-\Gamma_+\tau_1-\gamma_L\tau_2}\Big)\right]$$
$$-e^{-\Gamma(\tau_1+\tau_2)}\left[\frac{2(\alpha-a)}{\Delta m}\sin\big(\Delta m(\tau_1+\tau_2)\big)+\frac{4b}{\Delta m}\cos\big(\Delta m(\tau_1+\tau_2)\big)\right.$$
$$\left.-\left(\frac{4b}{\Delta m}-2e^{-(A-\gamma)(\tau_1+\tau_2)}-4\,\mathcal{R}e\Big[\epsilon_L+\epsilon_S-2y-\frac{4c}{\Delta\Gamma_-}\Big]\right)\cos\big(\Delta m(\tau_1-\tau_2)\big)\right]\Bigg\}$$
$$(B.4b)$$

$$\mathcal{G}(\ell^-,\tau_1;\ell^-,\tau_2)=\frac{|\mathcal{M}|^4}{8}\Bigg\{\frac{2\gamma}{\Delta\Gamma}\Big(e^{-\gamma_L(\tau_1+\tau_2)}-e^{-\gamma_S(\tau_1+\tau_2)}\Big)$$
$$\left[1-2\,\mathcal{R}e\Big(\epsilon_S+\epsilon_L-2y+\frac{4c}{\Delta\Gamma_-}\Big)\right]\Big(e^{-\gamma_S\tau_1-\gamma_L\tau_2}+e^{-\gamma_L\tau_1-\gamma_S\tau_2}\Big)$$
$$+\mathcal{R}e\left[\frac{8C}{\Delta\Gamma_+}\Big(e^{-\gamma_S\tau_1-\Gamma_+\tau_2}+e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\Big)\right]+\mathcal{R}e\left[\frac{8C}{\Delta\Gamma_-}\Big(e^{-\gamma_L\tau_1-\Gamma_+\tau_2}+e^{-\Gamma_+\tau_1-\gamma_L\tau_2}\Big)\right]$$
$$-e^{-\Gamma(\tau_1+\tau_2)}\left[\frac{2(\alpha-a)}{\Delta m}\sin\big(\Delta m(\tau_1+\tau_2)\big)+\frac{4b}{\Delta m}\cos\big(\Delta m(\tau_1+\tau_2)\big)\right.$$
$$\left.-\left(\frac{4b}{\Delta m}-2e^{-(A-\gamma)(\tau_1+\tau_2)}+4\,\mathcal{R}e\Big[\epsilon_L+\epsilon_S-2y-\frac{4c}{\Delta\Gamma_-}\Big]\right)\cos\big(\Delta m(\tau_1-\tau_2)\big)\right]\Bigg\}\ .$$
$$(B.4c)$$

APPENDIX C

As explained in the text, the asymmetry $\mathcal{A}_{\epsilon'}(\tau)$ in (4.19) allows the determination of real and imaginary parts of the ratio ϵ'/ϵ. To first order in this ratio, one finds:

$$\mathcal{A}_{\epsilon'}(\tau)=3\,\mathcal{R}e\Big(\frac{\epsilon'}{\epsilon}\Big)\ \frac{N_1(\tau)}{D(\tau)}-3\,\mathcal{I}m\Big(\frac{\epsilon'}{\epsilon}\Big)\ \frac{N_2(\tau)}{D(\tau)}\ . \qquad (C.1)$$

The explicit expression of the three coefficients N_1, N_2 and D is given below.

$$N_1(\tau)=e^{-\gamma_L\tau}\left[1+4\,\mathcal{R}e\Big(\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\Big)\right]-e^{-\gamma_S\tau}\left[1+4\,\mathcal{R}e\Big(\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\,\frac{\gamma_S-\Gamma_-}{\gamma_S+\Gamma_+}\Big)\right]$$

$$
- 4e^{-\Gamma\tau}\,\mathcal{R}e\biggl(e^{-i\Delta m\tau}\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\,\frac{2\Gamma}{\gamma_S+\Gamma_+}\biggr)\biggr]\ , \qquad (C.2)
$$

$$
\begin{aligned}
N_2(\tau) = 4e^{-\gamma_L\tau}\,\mathcal{I}m\biggl(\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\biggr) &- 4e^{-\gamma_S\tau}\,\mathcal{I}m\biggl(\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\,\frac{\gamma_S-\Gamma_-}{\gamma_S+\Gamma_+}\biggr)\\
&- 2e^{-\Gamma\tau}\,\mathcal{I}m\biggl[e^{-i\Delta m\tau}\biggl(1+\frac{2C}{\Delta\Gamma_+\,\eta^*_{+-}}\,\frac{2\Gamma}{\gamma_S+\Gamma_+}\biggr)\biggr]\ , \qquad (C.3)
\end{aligned}
$$

$$
\begin{aligned}
D(\tau) = e^{-\gamma_L\tau}&\biggl\{1+\frac{1}{|\eta_{+-}|^2}\biggl[\frac{\gamma}{\Delta\Gamma}-8\biggl|\frac{C}{\Delta\Gamma_+}\biggr|^2-4\,\mathcal{R}e\biggl(\frac{\epsilon_L C}{\Delta\Gamma}\biggr)\biggr]+8\,\mathcal{R}e\biggl(\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\biggr)\biggr\}\\
+\,e^{-\gamma_S\tau}&\biggl\{1-\frac{1}{|\eta_{+-}|^2}\biggl[\frac{\gamma_L}{\gamma_S}\biggl(\frac{\gamma}{\Delta\Gamma}-4\,\mathcal{R}e\biggl(\frac{\epsilon_L C}{\Delta\Gamma}\biggr)+8\biggl|\frac{C}{\Delta\Gamma_+}\biggr|^2\biggr)\biggr]\\
+\,8\,\mathcal{R}e&\biggl(\frac{C}{\Delta\Gamma_+\,\eta^*_{+-}}\,\frac{\gamma_S-\Gamma_-}{\gamma_S+\Gamma_+}\biggr)\biggr\}-2e^{-\Gamma\tau}\,\mathcal{R}e\biggl[e^{-i\Delta m\tau}\biggl(1+\frac{4C}{\Delta\Gamma_+\,\eta^*_{+-}}\,\frac{2\Gamma}{\gamma_S+\Gamma_+}\biggr)\biggr]\ .
\end{aligned} \qquad (C.4)
$$

APPENDIX D

As explained in the text, the radiative decay

$$
\phi\to\gamma\,K^0\overline{K^0}\ , \qquad (D.1)
$$

could give non-vanishing contributions to the correlations $\mathcal{G}(f_1,\tau_1;f_2,\tau_2)$. These background terms could mimic the effects of the non-standard parameters a, b, c, α, β, γ, making more problematic their experimental determinations. In this Appendix we collect the explicit expressions of the contributions $\widetilde{\mathcal{G}}$ to some of the observables $\mathcal{G}$ studied in Section 4, coming from the background process in $(D.1)$.

In the case of two charged pion final states, one gets, to second order in all small parameters:

$$
\begin{aligned}
\widetilde{\mathcal{G}}(\pi^+\pi^-,\tau_1;\pi^+\pi^-,\tau_2) = \frac{|X_{+-}|^4}{2}&\biggl\{e^{-\gamma_S(\tau_1+\tau_2)}\biggl\{1-\biggl(\frac{\gamma}{\Delta\Gamma}\biggr)^2\\
&+2\,\mathcal{R}e\biggl[2\biggl(\epsilon_L-\frac{2C^*}{\Delta\Gamma_-}+Y_{+-}\biggr)\biggl(\epsilon_S+\frac{2C}{\Delta\Gamma_-}\biggr)-\biggl(\epsilon_L-\frac{2C^*}{\Delta\Gamma_-}\biggr)^2\biggr]\biggr\}\\
&+e^{-\gamma_L(\tau_1+\tau_2)}\biggl(\frac{\gamma}{\Delta\Gamma}\biggr)^2-2\,\mathcal{R}e\biggl[e^{-\Gamma_+(\tau_1+\tau_2)}\biggl(\epsilon_L-\frac{2C^*}{\Delta\Gamma_-}+Y_{+-}\biggr)^2\biggr]\\
&+2\,\mathcal{R}e\biggl[\Bigl(e^{-\gamma_S\tau_1-\Gamma_+\tau_2}+e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\Bigr)\biggl(\epsilon_L-\epsilon_S-2\frac{C+C^*}{\Delta\Gamma_-}\biggr)\biggl(\epsilon_L-\frac{2C^*}{\Delta\Gamma_-}+Y_{+-}\biggr)\biggr]\biggr\}\ .
\end{aligned} \qquad (D.2)
$$

A similar expression holds for the correlation involving three pions in the final states; explicitly, one finds:

$$
\begin{aligned}
&\widetilde{\mathcal{G}}(\pi^+\pi^-\pi^0,\tau_1;\pi^+\pi^-\pi^0,\tau_2) = \frac{|X_{+-0}|^4}{2}\Bigg\{ e^{-\gamma_L(\tau_1+\tau_2)}\Bigg\{1-\left(\frac{\gamma}{\Delta\Gamma}\right)^2 \\
&+2\,\mathcal{R}e\left[2\left(\epsilon_S-\frac{2C}{\Delta\Gamma_-}+Y_{+-0}\right)\left(\epsilon_L+\frac{2C^*}{\Delta\Gamma_-}\right)-\left(\epsilon_S-\frac{2C}{\Delta\Gamma_-}\right)^2\right]\Bigg\} \\
&+e^{-\gamma_S(\tau_1+\tau_2)}\left(\frac{\gamma}{\Delta\Gamma}\right)^2-2\,\mathcal{R}e\left[e^{-\Gamma_-(\tau_1+\tau_2)}\left(\epsilon_S-\frac{2C}{\Delta\Gamma_-}+Y_{+-0}\right)^2\right] \\
&+2\,\mathcal{R}e\left[\left(e^{-\gamma_L\tau_1-\Gamma_-\tau_2}+e^{-\Gamma_-\tau_1-\gamma_L\tau_2}\right)\left(\epsilon_S-\epsilon_L-2\frac{C+C^*}{\Delta\Gamma_-}\right)\left(\epsilon_S-\frac{2C}{\Delta\Gamma_-}+Y_{+-0}\right)\right]\Bigg\}\ .
\end{aligned}
\tag{D.3}
$$

Finally, in the case of semileptonic final states, the contribution coming from the background process $(D.1)$ produces the term:

$$
\begin{aligned}
\widetilde{\mathcal{G}}(\ell^+,\tau_1;\ell^+,\tau_2) =&\frac{|\mathcal{M}|^4}{8}\Bigg\{e^{-\gamma_S(\tau_1+\tau_2)}\left[1-\frac{2\gamma}{\Delta\Gamma}+4\,\mathcal{R}e\left(\epsilon_S+x-y+\frac{2C}{\Delta\Gamma_-}\right)\right] \\
&+e^{-\gamma_L(\tau_1+\tau_2)}\left[1+\frac{2\gamma}{\Delta\Gamma}+4\,\mathcal{R}e\left(\epsilon_L-x-y+\frac{2C}{\Delta\Gamma_+}\right)\right] \\
&-2\,\mathcal{R}e\Bigg\{e^{-\Gamma_+(\tau_1+\tau_2)}\Bigg[1-4\Big(\mathcal{R}e(y)+i\mathcal{I}m(x)\Big)+2\left(\epsilon_L-\frac{2C^*}{\Delta\Gamma_-}\right) \\
&\qquad\qquad+2\left(\epsilon_S^*-\frac{2C^*}{\Delta\Gamma_+}\right)+2i\frac{B}{2\Delta m}\Bigg]\Bigg\} \\
&-2\,\mathcal{R}e\left[\left(e^{-\gamma_S\tau_1-\Gamma_+\tau_2}+e^{-\Gamma_+\tau_1-\gamma_S\tau_2}\right)\left(\epsilon_S-\epsilon_L+2\frac{C+C^*}{\Delta\Gamma_-}\right)\right] \\
&-2\,\mathcal{R}e\left[\left(e^{-\gamma_L\tau_1-\Gamma_+\tau_2}+e^{-\Gamma_+\tau_1-\gamma_L\tau_2}\right)\left(\epsilon_L^*-\epsilon_S^*+2\frac{C+C^*}{\Delta\Gamma_+}\right)\right] \\
&-2\left(e^{-\Gamma_+\tau_1-\Gamma_-\tau_2}+e^{-\Gamma_-\tau_1-\Gamma_+\tau_2}\right)\mathcal{I}m\left(\frac{B}{2\Delta m}\right)\Bigg\}\ .
\end{aligned}
\tag{D.4}
$$

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