Graded Problem Set 2

Date: 27.09.2013 Due date: 4.10.2013

Instructions : Please write the solution for each problem on a separate piece of paper. Sort these pieces of paper, with Problem 1 on top. Add this page as cover page, fill in your name, Sciper number, and the list of collaborators, and staple them all together on the left top.

Rules : You are allowed and encouraged to discuss these problems with your colleagues. However, each of you has to write down her solution in her own words. If you collaborated on a homework, write down the name of your collaborators and your sources in the space below. No points will be deducted for collaborations. But if we find similarities in solutions beyond the listed collaborations we will consider it as cheating. Please note that EPFL has a VERY strict policy on cheating and you might be in BIG trouble. It is simply not worth it.

Grading : Each problem is worth 20 points. You will get 10 bonus points if you solve the last problem (i.e., Problem 6).

Collaborators and sources :

Name : ....

Sciper : ....

Problem 1	... / 20
Problem 2	... / 20
Problem 3	... / 20
Problem 4	... / 15
Problem 5	... / 25
Problem 6	+ ... / 10
TOTAL	... / 100

Problem 1. Show the following :

a) $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent using a truth table.

b) $(p \to q) \wedge (\neg p \to (\neg q \to (p \wedge \neg p)))$ and q are logically equivalent using the laws of logical equivalence.

Problem 2. Suppose $P(x, y)$ is a predicate and let the universe for the variables x and y be $\{1, 2, 3\}$. Suppose that $P(1, 3), P(2, 1), P(2, 2), P(2, 3), P(3, 1), P(3, 2)$ are true, and that $P(x, y)$ is false otherwise. Determine whether the following statements are true, justifying your answer.

1. $\forall x \exists y P(x, y)$
2. $\exists x \forall y P(x, y)$
3. $\neg \exists x \exists y (P(x, y) \wedge \neg P(y, x))$
4. $\forall y \exists x (P(x, y) \to P(y, x))$
5. $\forall x \forall y (x \neq y \to (P(x, y) \vee P(y, x)))$
6. $\forall y \exists x (x \leq y \wedge P(x, y))$

Problem 3. Let $F(A)$ be the predicate "A is a finite set" and let $S(A, B)$ be the predicate "A is contained in B". Suppose the universe of discourse consists of all sets. Translate the following statements into symbols. Please notice that the solution might not be unique, because there might be many ways of expressing the statements. You are required to write only one possible solution.

1. Not all sets are finite.
2. Every subset of a finite set is finite.
3. No infinite set is contained in a finite set.

Problem 4. Show that the premises "Everyone who read the textbook passed the exam", and "Ed read the textbook" imply the conclusion "Ed passed the exam".

Problem 5. From the following premises : "Someone has a Bachelor degree in Mathematics and has less than 125 credits", "All the students who obtain a bachelor degree in Mathematics have passed the exam *Analysis II*" and "All the students who have passed the exam *Analysis I* have obtained at least 125 credits" prove the conclusion "Someone has passed the exam *Analysis II* and has not passed *Analysis I*". You are not required to mention explicitly the usage of the commutative law.

Problem 6. (Extra credit) Each woman of the town of Questionland always lies or always gives the correct reply. A woman of Questionland will give only a "Yes" or a "No" response to a question you ask her. Suppose you are visiting this area and you arrive to a fork in the road. One branch leads to the city of Questionland, and the other branch leads deep onto the jungle. A woman of Questionland is standing at the fork in the road. What one question can you ask her to determine which branch to take?