Graph Theory Applications — EPFL, Spring 2014

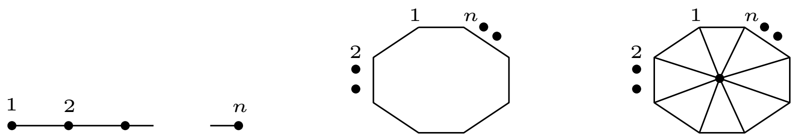
Problem Set 8

Date: 10.04.2014 — Not graded

The edge chromatic number of a graph G, namely $\chi'(G)$ is the fewest number of colors necessary to color each edge of G such that no two edges incident on the same vertex have the same color.

Recall that, given a graph G, its largest vertex degree is denoted by $\Delta(G)$.

Problem 1. Let $n \geq 3$. What is χ' for the following graphs?



(a) Path Graph P_n (b) Cycle Graph C_n (c) Wheel Graph W_{n+1}

Problem 2. Show that for a nonempty simple regular graph G with odd number of vertices $\chi'(G) \geq \Delta(G) + 1$.

Problem 3. Let m^* be the size of the maximum matching of a graph with m edges. Then, prove that

$$\chi' \geq \left\lceil \frac{m}{m^*} \right\rceil .$$

Problem 4. Let $K_{m,n}$ denote the simple bipartite graph with bipartition (X, Y) s.t. $|X| = m$, $|Y| = n$, and for any $x \in X$ and $y \in Y$ $\deg(x) = n$ and $\deg(y) = m$. Prove, by finding an appropriate edge coloring, that $\chi'(K_{m,n}) = \Delta(K_{m,n})$.

Problem 5. Let G be a 3-regular graph with $\chi' = 4$. Prove that G is not Hamiltonian.