

## Problem Set 6

Date: 27.03.2014

Not graded

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**Problem 1.** What is the length of the maximum matching in the cycle graph on  $n$  vertices? Can you give a closed form expression?

**Problem 2.** Show that the *cube* (defined in Problem Set 3) has a perfect matching.

**Problem 3.** Show that a tree cannot have two distinct perfect matchings. (Two matchings are distinct if there exists an edge that is contained in one matching but not the other.)

**Problem 4.** Two people play a game on a graph  $G$  by alternately selecting distinct vertices  $v_1, v_2, v_3, \dots$  such that for  $i > 0$ ,  $v_i$  is adjacent to  $v_{i-1}$ . The last player who is able to select a vertex wins. If player 1 is the first to choose a vertex, show that  $G$  has a perfect matching if and only if there is a winning strategy for player 2.