Problem Set 3

Date: 6.03.2014 Not graded

Problem 1. We define the *complement* G^c of a graph G as the graph on the same vertex set as G, in which two vertices are joined by an edge if and only if they are not joined by an edge in G. Prove that it cannot happen that both G and G^c are disconnected.

Problem 2. A permutation matrix is a matrix obtained by permuting the rows of the $n \times n$ identity matrix according to some permutation π of the numbers from 1 to n.

Let P be a permutation matrix. Show that if A_G is the adjacency matrix of the graph, then $A_H = P^T A_G P$ is an adjacency matrix corresponding to the same graph with the vertices renumbered.

Problem 3. Let G be a regular graph with degree k. Show that k is an eigenvalue of the adjacency matrix A of the graph G.

Problem 4. Continuing with the above problem, show that for a k-regular connected graph G, $-k$ is also an eigenvalue of A if and only if G is bipartite.

Problem 5. We call *cube* the graph G_{C} whose vertex set V_{C} is the set $\{0, 1\}^n$ of all n-dimensional $0 - 1$ vectors and in which two vertices form an edge if their corresponding vectors differ in exactly one component. Show that the *cube* is bipartite.

Problem 6. Let A be an 3×3 matrix with non-negative entries. Using the Brouwer fixed point theorem, prove that A must have an eigenvector with non-negative coefficients. *Hint. Consider the set of vectors whose components are non-negative and sum to* 1.