Problem Set 10

Date: 08.05.2014 Not graded

Problem 1. You have been given the task of preparing a *Mega Problem Set* for the *Graph Theory Applications* course with 100 questions. The questions must come from 10 different categories, 10 for each category. To assist you, you can pick the questions from a book containing exactly 100 questions, but each question has several categories attached to it. Formulate a flow problem to check whether it is possible to get all the questions you want from the book.

Problem 2. Show how to transform the following types of flow problems to a standard flow problem.

1. Each vertex in the network has a capacity and the flow passing through the vertex cannot exceed this number.
2. Each edge has a lower bound on the flow that passes through it. Note that edge capacities are upper bounds on the flow.

Problem 3. In a public building such as a movie theater, it is important to have a plan of exit in the event of a fire. We will design such an emergency exit plan using max-flows. Suppose a movie theater is represented by a graph $G = (V; E)$, where each room, landing, or other location is represented by a vertex and each corridor or stairway is represented by an edge. Each corridor has the same capacity c, meaning that at most c people can pass through the corridor at once. Traversing a corridor from one end to the other takes one time step and traversing a room takes zero time steps.

1. Suppose all people are initially in a single room s, and there is a single exit t. Show how to use maximum flow to find a fastest way to get everyone out of the building.

   Hint 1. Create another graph that has vertices to represent each room at each time step.

   Hint 2. Let M be the number of people to be moved out. What is a (trivial) lower bound on the number of time steps required to bring people out of the movie theater? What is an upper bound on this quantity? Let k between the difference between the upper bound and the lower bound. Using *Hint 1*, provide an algorithm to decide if all the people can be moved out in T steps. Then, how can you employ that algorithm $O(\log k)$ times to find the shortest time in which all the people can be moved out?

2. Show how the same idea can be used when people are initially in multiple locations and there are multiple exits.
3. Finally, suppose that it takes different (but integer) amounts of time to cross different corridors or stairways, and that for each such corridor or stairway e, you are also given an integer $t(e)$ which is the number of seconds required to cross e. Now show how to transform your algorithm in point 1. to find a fastest way to get everyone out of the building.

Problem 4. Let G be a k-connected graph, let x be a vertex of G, and let $Y \subseteq V \setminus \{x\}$ be a set of at least k vertices of G. Then, there exists a family of k internally disjoint (x, Y)-paths whose terminal vertices are distinct.