Exercises november 16, 2007. Quantum information theory and computation

Exercise 1. Example of comparison between Von Neumann and Shannon entropies

Suppose $\rho = p|0\rangle\langle 0| + \frac{(1-p)}{2}(|0\rangle + |1\rangle)(\langle 0| + \langle 1|)$. Evaluate $S(\rho)$ and compare with the classical Shannon entropy corresponding to $\{p, 1-p\}$.

Exercise 2. Von Neumann Entropy of a tensor product

Consider a composite system with tensor product density matrix $\rho \otimes \sigma$. Prove that

$$S(\rho \otimes \sigma) = S(\rho) + S(\sigma)$$

Exercise 3. Entanglement and negative conditional entropy

Consider a pure state $|AB\rangle$ of a composite system (say shared by Alice and Bob). Prove that $|AB\rangle$ is entangled if and only if the conditional Von Neumann entropy is strictly *negative*.

Exercise 4. Analog of Araki-Lieb inequality for conditional entropy

First show that for three random variables with any joint distribution the Shannon entropy always satisfies

$$H(X, Y|Z) \geq H(X|Z)$$

Consider now a tripartite quantum system with Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B \otimes \mathcal{H}_C$. Show that for quantum entropies it is *not* always true that $S(A, B|C) \geq S(A|C)$. Prove that instead the following is always true

$$S(A, B|C) \geq |S(A|C) - S(B|C)|$$