Exercises october 19, 2007. Quantum information theory and computation

Exercise 1. Entanglement of Bell states

Prove that the four Bell states belonging to the Hilbert space $\mathbf{C}^2 \otimes \mathbf{C}^2$ are entangled. In other words you have to show that it is not possible to find $|\phi\rangle \in \mathbf{C}^2$ and $|\psi\rangle \in \mathbf{C}^2$ such that a Bell state equals $|\phi\rangle \otimes |\psi\rangle$.

Exercise 2. Three particle entanglement - GHZ states

Find the simplest three Qbit *fully entangled* state you can think of. Here fully entangled means that it cannot be written as the tensor product of three one Qbit states *and* it cannot be written as the tensor product of a two with a one Qbit state *and* it cannot be written as a tensor product of a one with a two Qbit state. Think of a way of generating such a state from tensor product state $|000\rangle$ by a unitary operation and give the corresponding quantum circuit. Find an orthonormal basis of entangled states for the Hilbert space $\mathbf{C}^2 \otimes \mathbf{C}^2 \otimes \mathbf{C}^2$.

The simplest such states are called GHZ states after Greene, Horne and Zeilinger. They can be produced and manipulated experimentally.

Exercise 3. Entanglement purification

The purpose of the exercise is to show from non-fully entangled states we can create with finite probability fully entangled states. We will come back to this later in the course when we will treat "entanglement purification". We have four Qbits in the state $|\Psi_\alpha\rangle \otimes |\Psi_\alpha\rangle$ where

$$|\Psi_\alpha\rangle = \alpha|00\rangle + (1-\alpha^2)^{1/2}|11\rangle$$

We measure the observable

$$Z \otimes I \otimes I \otimes I + I \otimes I \otimes Z \otimes I$$

Give all the possible outcomes of this measurement together with their respective probabilities. What is the probability that we obtain a fully entangled state ?

Hint: write the observable in the Dirac notation.