Discrete Structures EPFL, Fall 2013

Graded Problem Set 10

Date: 22.11.2013 Due date: 29.11.2013, noon

Instructions: Please write the solution for each problem on a separate piece of paper. Sort these pieces of paper, with Problem 1 on top. Add this page as cover page, fill in your name, Sciper number, and the list of collaborators, and staple them all together on the left top.

Rules: You are allowed and encouraged to discuss these problems with your colleagues. However, each of you has to write down her solution in her own words. If you collaborated on a homework, write down the name of your collaborators and your sources in the space below. No points will be deducted for collaborations. But if we find similarities in solutions beyond the listed collaborations we will consider it as cheating. Please note that EPFL has a VERY strict policy on cheating and you might be in BIG trouble. It is simply not worth it.

Grading: Each problem is worth 20 points.

Collaborators and sources:

First name: ....

Last name: ....

Sciper: ....

Problem 1 ... / 20

Problem 2 ... / 20

Problem 3 ... / 20

Problem 4 ... / 20

Problem 5 ... / 20

Problem 6 ... / 20

TOTAL ... / 120

Problem 1.

(a) Suppose a restaurant serves a "special dinner" consisting of a soup, salad, entree, dessert, and beverage. The restaurant has five kinds of soup, three kinds of salad, ten entrees, five desserts , and four beverages. How many different special dinners are possible ? (Two special dinners are different if they differ in at least one selection.)

(b) How many different strings can be made using all the letters in the word GOOGOL ?

(c) A club with 20 women and 17 men needs to form a committee of size six.

i) How many committees are possible ?

ii) How many committees are possible if the committee must have three women and three men ?

iii) How many committees are possible if the committee must have at least two men ?

iv) How many committees are possible if the committee must consist of all women or all men ?

(d) How many logical propositions are there with the k distinct variables $x_1, x_2, ..., x_k$? You should count propositions that are logically equivalent (i.e. have the same truth table) only once.

Problem 2. Here is an incorrect solution to a problem. Find the error, explain why it is not correct, and give the correct answer.

Problem: Find the number of ways to get two pairs of two different ranks (such that 2 jacks and 2 fives) in a 4-card hand from an ordinary deck of 52 cards.

Solution: There are 13 ways to get a rank (such as "kings") for the first pair and $\binom{4}{2}$ ways to get a pair of that eank. Similarly, there are 12 ways to get a rank (such as "sevens") for the second pair and $\binom{4}{2}$ ways to get a pair of that rank. Therefore there are $13 \cdot 12 \cdot \binom{4}{2} \cdot \binom{4}{2}$ ways to get 2 pairs.

Problem 3.

(a) A factory makes automobile parts. Each part has a code consisting of a letter and three digits, such as C117, O076, or Z920. Last week the factory made 60,000 parts. Prove that there are at least three parts that have the same serial number.

(b) Prove or disprove that if five points are picked on or in the interior of a square of side length 2, then there are at least two of these points no farther than $\sqrt{2}$ apart.

Problem 4. Let n be a positive integer. Show that the number of binary strings of length n with an even number of 1s is equal to the number of binary strings with an odd number of 1s.

Problem 5.

(a) Let $\mathcal{F}$ be the set of increasing functions $f : \{1, \cdots, k\} \to \{1, \cdots, n\}$, i.e., $f(i+1) > f(i)$ for any $i \in \{1, \cdots, k-1\}$ (fr: *f strictement croissante*). Compute $|\mathcal{F}|$.

(b) Let $\mathcal{G}$ be the set of non-decreasing functions $g : \{1, \cdots, k\} \to \{1, \cdots, n\}$, i.e., $g(i+1) \geq g(i)$ for any $i \in \{1, \cdots, k-1\}$ (fr: *g [monotone] croissante*). Compute $|\mathcal{G}|$.

Problem 6. Find the number of matrices with m rows and n columns whose elements are in $\{-1, +1\}$ s.t. the product of the elements of each row and the product of the elements of each column is -1.