Discrete Structures — EPFL, Fall 2013

Problem Set 5

Date: 18.10.2013 — Not graded

Problem 1. Suppose that you want to sort the integers $a_1, a_2, \ldots, a_k$ in increasing order with **BubbleSort**. Recall that the algorithm **BubbleSort** rearranges the elements in increasing order by successively sweeping the list from left to right, comparing adjacent numbers and interchanging them if they are in the wrong order. What is the initial ordering of $a_1, a_2, \ldots, a_k$ that maximizes the number of required swaps?
Write also the number of swaps needed and the best big-O approximation.

Problem 2. Consider the following algorithms:

Algorithm 1

```
Require: n: positive integer
m ← 1
j ← 1
while j ≤ n do
    k ← 1
    while k ≤ m do
        print “Discrete Structures”
        k ← k + 1
    m ← 2 · m
    j ← j + 1
```

Algorithm 2

```
Require: m: real number
while m ≥ e or m < 0 do
    m ← (2/3) m
    print “Hello world”
```

a) Does **Algorithm 1** always terminate? How about **Algorithm 2**? If not, under what conditions do they terminate?

b) How many times does **Algorithm 1** print “Discrete Structures” as a function of n?
Same question about **Algorithm 2** printing “Hello world” as a function of m. You might need to use truncation functions (like $\lfloor \cdot \rfloor$ and $\lceil \cdot \rceil$).

Problem 3. Find the best big-O function for the following functions, choosing the answer among 1, $\log_2 n$, n, $n \log_2 n$, n^2, n^3, n^4, n^5, 2^n, $n!$.

a) $f(n) = 1 + 4 + 7 + \cdots + (3n + 1)$

b) $g(n) = 1 + 3 + 5 + 7 + \cdots + (2n - 1)$

c) $\dfrac{3 - 2n^4 - 4n}{2n^3 - 3n}$

d) $h(n) = 1 + 2 + 3 + \cdots + (n^2 - 1) + n^2$

e) $\lceil n + 2 \rceil \cdot \lceil n/3 \rceil$

f) $3n^4 + \log_2 n^8$

Problem 4. Arrange the following functions in a list, so that each is big-O of the next one in the list:

$n \cdot 2^n$	$-\dfrac{1}{\sqrt{n}}$	$n \log n$	$n^{\log n}$	n^{-e^4}	$0,0097^n$	e^{2n}
2.0001^n	2^n	$\log \log \log n$	n^{2013}	$\left(-\dfrac{1}{2}\right)^n$	42	n
$\dfrac{1}{n^n}$	$(-3)^n$	$n \log \log n$	$n!$	e^n	$e^{\sqrt{n}}$	

Problem 5. Use the definition of big-O to prove that:

1. $\dfrac{3n - 8 - 4n^3}{2n - 1}$ is $O(n^2)$.
2. $1^3 + 2^3 + 3^3 + \cdots + n^3$ is $O(n^4)$.
3. $4^n + n2^n$ is $O(4^n)$.