Discrete Structures EPFL, Fall 2013

Graded Problem Set 4

Date: 11.10.2013 Due date: 18.10.2013

Instructions: Please write the solution for each problem on a separate piece of paper. Sort these pieces of paper, with Problem 1 on top. Add this page as cover page, fill in your name, Sciper number, and the list of collaborators, and staple them all together on the left top.

Rules: You are allowed and encouraged to discuss these problems with your colleagues. However, each of you has to write down her solution in her own words. If you collaborated on a homework, write down the name of your collaborators and your sources in the space below. No points will be deducted for collaborations. But if we find similarities in solutions beyond the listed collaborations we will consider it as cheating. Please note that EPFL has a VERY strict policy on cheating and you might be in BIG trouble. It is simply not worth it.

Grading: Each problem is worth 20 points. You will get 10 bonus points if you solve the last problem (i.e., Problem 6).

Collaborators and sources:

Name : ....

Sciper : ....

Problem 1 ... / 20

Problem 2 ... / 20

Problem 3 ... / 20

Problem 4 ... / 20

Problem 5 ... / 20

Problem 6 + ... / 10

TOTAL ... / 100

Problem 1. You have a salary of 40000 CHF per year.

(a) How much will you earn in n years if you receive a raise of 3% per year?

(b) How much will you earn in n years if you receive a raise of 5% per year?

(c) How much will you earn in n years if you receive a raise of 1000 CHF and additionally 2% of your previous year's salary?

Problem 2. In a remote island of the Pacific, there is a population of 10000 people. According to the population model of the scientist Prof. Crazytheory, next year the number of births will exceed the number of deaths by 98 units. Prof. Crazytheory also states that the difference between births and deaths is going to increase by 1 unit every year.

According to this model, in how many years is the population going to exceed the value of 20000 units?

Problem 3. Let $\mathbb{N}$ be the set of natural numbers including 0 and let $A = \mathbb{N} \cup \{e, e^e, e^{e^e}, e^{e^{e^e}}\}$. Show that $|A| = |\mathbb{N}|$.

Problem 4.

(a) Prove that $|(-1,1)| = |\mathbb{R}|$.

Hint: one of the functions seen in the previous problem set might help you...

(b) Show that $|(0,1)| = |(0,1]|$ by displaying a bijective function $f : (0,1) \to (0,1]$. Justify your answer.

Hint: consider a countable subset A of $(0,1]$, for example $A = \{1/n : n \in \mathbb{Z}, n \geq 1\}$, and recall Hilbert's Grand Hotel...

Problem 5. Let C be the set of strictly increasing functions whose domain and codomain is a set of integers of cardinality 2013. Show that $|C| = |\mathcal{P}(\emptyset)|$.

Hint: pigeonhole...

Problem 6. (Extra credit) Let $\mathcal{F}$ be the set of functions from $\mathbb{N}$ to $\mathbb{N}$ and $\mathcal{G}$ be the set of functions from $\mathbb{N}$ to $\{0,1\}$. Prove that $|\mathcal{F}| = |\mathcal{G}|$.

Hint: show that there is an injective function $f : \mathcal{F} \to \mathcal{G}$, i.e., a function which encodes infinite sequences of natural numbers into infinite binary strings unequivocally.