Discrete Structures EPFL, Fall 2013

Problem Set 3

Date: 4.10.2013 Not graded

In the problems that follow we denote by $\mathbb{N}$ the set of natural numbers including 0.

Problem 1. Prove or disprove:

a) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ by logical equivalence.

b) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$ by Venn diagram.

Problem 2. Let A and B be disjoint sets such that $|A| = n$ and $|B| = m$. Find the cardinalities of the following sets:

a) $A \times B$

b) $\mathcal{P}(B)$

c) $\mathcal{P}(\,\mathcal{P}(A) \times B\,)$

d) $\mathcal{P}(A \cup B)$

e) $A \times \mathcal{P}(\,\mathcal{P}(A \cap B)\,)$

f) $(A \setminus B) \times (B \setminus A)$

Problem 3. Determine whether the given set is the power set of some set. If the set is a power set, give the set of which it is a power set.

a) $\{\varnothing,\ \{\varnothing\},\ \{a\},\ \{\{a\}\},\ \{\{\{a\}\}\},\ \{\varnothing, a\},\ \{\varnothing, \{a\}\},\ \{\varnothing, \{\{a\}\}\},\ \{a, \{a\}\},\ \{a, \{\{a\}\}\},\ \{\{a\}, \{\{a\}\}\},$
$\{\varnothing, a, \{a\}\},\ \{\varnothing, a, \{\{a\}\}\},\ \{\varnothing, \{a\}, \{\{a\}\}\},\ \{a, \{a\}, \{\{a\}\}\},\ \{\varnothing, a, \{a\}, \{\{a\}\}\}.$

b) $\{\varnothing, \{a\}\}$.

c) $\{\varnothing, \{a\}, \{\varnothing, a\}\}$.

d) $\{\varnothing, \{a\}, \{\varnothing\}, \{a, \varnothing\}\}$.

e) $\{\varnothing, \{a, \varnothing\}\}$.

Problem 4. At a dinner there are 35 people. You know that 18 of them are *mathematicians*, 13 of them are *poets* and 14 of them are *painters*. All the 35 people fit in **at least** one of these categories. In addition, there are 7 people who are poets **and** painters and all the people who are mathematicians **and** painters are also poets. Find the number of people who are mathematicians **and** poets.

Problem 5. Determine whether the rule describes a function with the given domain and codomain.

a) $f : \mathbb{N} \to \mathbb{N}$ where $f(n) = \sqrt{n}$.

b) $h : \mathbb{R} \to \mathbb{R}$ where $h(x) = \sqrt{x}$.

c) $g : \mathbb{N} \to \mathbb{N}$ where $g(n) =$ any integer $> n$.

d) $F : \mathbb{R} \to \mathbb{R}$ where $F(x) = \dfrac{1}{x-5}$.

e) $F : \mathbb{Z} \to \mathbb{R}$ where $F(x) = \dfrac{1}{x^2-5}$.

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g) $G : \mathbb{R} \to \mathbb{R}$ where $G(x) = \begin{cases} x+2 & \text{if } x \geq 0 \\ x-1 & \text{if } x \leq 4. \end{cases}$

h) $f : \mathbb{R} \to \mathbb{R}$ where $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ x-1 & \text{if } x \geq 4. \end{cases}$

i) $G : \mathbb{Q} \to \mathbb{Q}$ where $G(p/q) = q$.

Problem 6. Find if the following functions are injective, surjective, bijective or none of the previous:

a) $f : \mathbb{N} \to \mathbb{N}$ where $f(n) = 4\,n + 1$.

b) $f : \mathbb{N} \to \mathbb{E}$ where $f(n) = 2\,n$ and $\mathbb{E}$ is the set of nonnegative even numbers.

c) $f : \mathbb{R} \to (-1, 1)$ where $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

d) $f : \mathbb{R} \to \{z \in \mathbb{C} : |z| = 1\}$ where $f(x) = e^{ix}$.

e) $f : \mathbb{Z} \to \mathbb{R}$ where $f(x) = \lfloor x/2 \rfloor$.

Problem 7. Suppose $g : A \to B$ and $f : B \to C$ where $A = \{1, 2, 3, 4\}$, $B = \{a, b, c\}$, $C = \{2, 8, 10\}$, and g and f are defined by $g = \{(1, b), (2, a), (3, b), (4, a)\}$ and $f = \{(a, 8), (b, 10), (c, 2)\}$.

1. Find $f \circ g$.
2. Find f^{-1}.
3. Find $f \circ f^{-1}$.
4. Explain why g^{-1} is not a function.