Discrete Structures EPFL, Fall 2013

Problem Set 1

Date: 20.09.2013 Not graded

Problem 1. There are two types of people on an island: knights and knaves. Knights always tell the truth. Knaves always lie. A says: "B is a knight." B says: "The two of us are of opposite types." Determine the types of A and B.

Problem 2. Determine the truth value of the following propositions.

a) $1 + 1 = 3$ if and only if $2 + 2 = 3$.

b) If it is raining, then it is raining.

c) If $1 < 0$, then $3 = 4$.

d) If $2 + 1 = 3$, then $2 = 3 - 1$.

e) If $1 + 1 = 2$ or $1 + 1 = 3$, then $2 + 2 = 3$ and $2 + 2 = 4$.

Problem 3. Consider the proposition $(p \to (q \wedge r)) \wedge ((\neg p) \to (q \vee r))$, depending on propositional variables p, q and r.

a) Write down the truth table for this proposition.

b) If a fair coin is flipped to determine the truth value of each of the variables p, q and r, how likely is it that the whole proposition is true? How likely is it that the proposition is false?

Problem 4. Find a proposition with three variables p, q, and r that

a) is true when p and r are true and q is false, and false otherwise;

b) is true when at most one of the three variables is true, and false otherwise;

c) is never true.

Note that there are many possible solutions to this problem.

Problem 5. A set of propositions is *consistent* if there is an assignment of truth values to each of the variables in the propositions that makes each proposition true. Is the following set of propositions consistent? It may help to write the propositions using logic symbols.

- The system is in multiuser state if and only if it is operating normally.
- If the system is operating normally, the kernel is functioning.
- The kernel is not functioning or the system is in interrupt mode.
- If the system is not in multiuser state, then it is in interrupt mode.
- The system is in interrupt mode.

Problem 6. In this problem you will show that $((p \to q) \wedge (q \to r)) \to (p \to r)$ is a tautology by proceeding in steps, without ever writing down the truth tables.

a) First show that $p \vee (\neg p \wedge q)$ is logically equivalent to $p \vee q$ by first applying the distribution law of $\vee$ with respect to $\wedge$, then the negation law and the identity law.

b) Take the proposition $((p \to q) \wedge (q \to r)) \to (p \to r)$ and transform it to an equivalent form containing only conjunctions, disjunctions and negations by replacing any implication of the form $s \to t$ with its definition $\neg s \vee t$.

c) Apply the De Morgan's law and the distributive laws as many times as needed until you reach $(p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r$.

d) Use now point a) to simplify the expressions $\neg p \vee (p \wedge \neg q)$ and $r \vee (\neg r \wedge q)$ that appear above. Note that you need to use commutativity and associativity as well to achieve this. [1]

e) Conclude that the formula at hand is a tautology by using the negation and domination laws successively.

Problem 7. (Extra) You are given a list of statements numbered from 1 to 50. The n-th statement reads "exactly n of the statements in this list are false." Which statements are true and which are false? What if we replace "exactly" by "at least"?

[1]Normally we do these transformations without thinking too much but it helps to be aware of everything that goes on.