Solution to Problem Set 1

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Problem 1. Suppose A is knight; then B must be a knight, because A tells the truth; but then A and B are of the same type, so B lies: contradiction! Now if A is not a knight, then A lies and B is not a knight either.

Problem 2.

a) True: $1 + 1 = 3$ if and only if $2 + 2 = 3$.

b) True: If it is raining, then it is raining.

c) True: If $1 < 0$, then $3 = 4$.

d) True: If $2 + 1 = 3$, then $2 = 3 - 1$.

e) False: If $1 + 1 = 2$ or $1 + 1 = 3$, then $2 + 2 = 3$ and $2 + 2 = 4$.

Problem 3.

a)

p	q	r	Value
F	F	F	F
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	F
T	T	T	T

b) If the three coins are fair, each line in the table is equally likely to occur. Out of the 8 possible cases, 4 make the proposition true and the other 4 make the proposition false. Thus, the probabilities of the proposition being true and false are both 50%.

Problem 4. There are many possible solutions. For example:

a) $p \wedge \neg q \wedge r$;

b) $\neg((p \wedge q) \vee (q \wedge r) \vee (p \wedge r))$;

c) $(p \wedge \neg p) \wedge q \wedge r$.

Problem 5. Let p = "system in multiuser state", q = "system operating normally", r = "kernel functioning", s = "system in interrupt mode".

The propositions can then be rewritten as:

1. $p \iff q$,

2. $q \to r$,

3. $\neg r \vee s$,

4. $\neg p \to s$,

5. s.

To satisfy 5, we need that s is true, which also ensures that 3 and 4 are satisfied. For the first proposition, we need p and q to have the same truth value. In case both p and q are true, r needs to be also true to satisfy 2, otherwise r can take any value.

We have thus found many (in fact all) satisfying truth assignments, so the set of propositions is indeed consistent.

Problem 6. Apply rules successively:

$$\begin{aligned} & p \vee (\neg p \wedge q) & \\ \equiv\ & (p \vee \neg p) \wedge (p \vee q) & \text{(distributivity)} \\ \equiv\ & \mathrm{T} \wedge (p \vee q) & \text{(negation law)} \\ \equiv\ & (p \vee q) & \text{(identity law);} \end{aligned}$$

$$\begin{aligned} & (p \to q) \wedge (q \to r) \to (p \to r) & \\ \equiv\ & \neg((\neg p \vee q) \wedge (\neg q \vee r)) \vee (\neg p \vee r) & \text{definition of implication} \\ \equiv\ & (\neg(\neg p \vee q) \vee \neg(\neg q \vee r)) \vee \neg p \vee r & \text{De Morgan, associativity} \\ \equiv\ & (\neg\neg p \wedge \neg q) \vee (\neg\neg q \wedge \neg r) \vee \neg p \vee r & \text{De Morgan, associativity} \\ \equiv\ & (p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r & \text{double negation law} \\ \equiv\ & \neg p \vee (p \wedge \neg q) \vee r \vee (q \wedge \neg r) & \text{commutativity} \\ \equiv\ & \neg p \vee \neg q \vee r \vee q & \text{point a), see above, applied twice} \\ \equiv\ & q \vee \neg q \vee \neg p \vee r & \text{commutativity} \\ \equiv\ & \mathrm{T} \vee \neg p \vee r & \text{negation law} \\ \equiv\ & \mathrm{T} & \text{domination law.} \end{aligned}$$

Problem 7. Clearly there is at most one statement that is correct. If there is no correct statement, then 50 statements are false and thus the 50th statement is correct: contradiction! If there is exactly one correct statement, 49 statements are false. Thus the 49th statement is correct (and the rest are false).

If we replace "exactly" by "at least", notice that if the $n+1$-th statement is true, so is the n-th and as a consequence all the statements that come before it. Suppose exactly the first m statements are true. Then there are $50 - m$ false statements, which makes the first $50 - m$ statements true. This implies $50 - m = m$, and so $m = 25$.