Handout 10 Statistical Physics for Communication and Computer Science
Homework 10 May 1, 2013

Problem 1 (Statistics of AMP and IST un-threshoded estimates)**.** Consider a sparse signal $\underline{x}_0$ with n iid components distributed as $(1-\epsilon)\delta(x_0) + \frac{\epsilon}{2}\delta(x-1) + \frac{\epsilon}{2}\delta(x+1)$. Generate m noisy measurements $\underline{y} = \frac{1}{\sqrt{m}}\tilde{A}\underline{x} + \underline{z}$ where $\tilde{A}_{ai}$ are iid uniform in $\{+1,-1\}$ and z_a are iid Gaussian zero mean and variance σ^2.

Consider the AMP iterations

$$\begin{cases} \hat{x}_i^{(t+1)} = \eta(\hat{x}_i^{(t)} + \frac{1}{\sqrt{m}}\sum_{b=1}^m \tilde{A}_{bi} r_b^t; \theta^{(t)}), \\ r_a^{(t)} = y_a - \frac{1}{\sqrt{m}}\sum_{j=1}^n \tilde{A}_{aj}\hat{x}_j^{(t-1)} + r_a^{t-1}\frac{\|\hat{x}^{(t)}\|_0}{m}, \end{cases}$$

with the choice $\theta^{(t)} = \alpha\|\underline{r}^{(t)}\|_2/\sqrt{m}$. In class we derived through heuristic means that the i-th component, given $\underline{x}_0$, of the un-thresholded estimate

$$\hat{x}_i^{(t)} + \frac{1}{\sqrt{m}}\sum_{b=1}^m \tilde{A}_{bi} r_b^{(t)},$$

has Gaussian statistics. The mean is x_{0i} and the variance $\sigma^2 + (\tilde{\tau})^{(2)}$ where $(\tilde{\tau})^{(2)} = \|\underline{x}^{(t)} - \underline{x}_0\|_2^2/n$.

Perform an experiment to check this numerically. Compute also the statistics of the un-thresholded estimate for the IST iterations, i.e. when the Onsager term $r_a^{t-1}\frac{\|\hat{x}^{(t)}\|_0}{m}$ is removed. Compare the two histograms.

Indications: Fix a signal realization $\underline{x}_0$. Try $n = 4000$, $m = 2000$, $\epsilon = 0.125$ and 40 instances for A and $\underline{z}$. Try various values for σ and α. Look at the i-th components of the un-thresholded estimate for components such that say $x_{0i} = +1$ (or -1, or 0).