

# ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

**Handout 9**  
Homework 9

Statistical Physics for Communication and Computer Science  
April 21, 2011, INR 113 - 9:15-11:00

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**Problem 1** (Preferential Attachment). The purpose of this homework is to use the Wormald method to study a model for "preferential attachment." Consider  $n$  nodes. Initially all nodes have degree 0. Assume that we allow a maximum degree of  $d_{\max}$ . We proceed as follows. At every step pick two nodes from the set of all nodes which have degree at most  $d_{\max} - 1$ . Rather than picking them with uniform probability pick them proportional to their current degree. More precisely, assume that at time  $t$  you have  $D_i(t)$  nodes of degree  $i$ . Then pick a node of degree  $i$  with probability

$$\begin{cases} \frac{D_i(t)}{\sum_{j=0}^{d_{\max}-1} D_j(t)}, & 0 \leq i < d_{\max}, \\ 0, & i = d_{\max}. \end{cases}$$

Initially, we have  $D_0(t=0) = n$  and  $D_i(t=0) = 0$  for  $i = 1, \dots, d_{\max}$ .

Note that at time  $t = nd_{\max}/2$ , all nodes will have maximum degree. Pick  $d_{\max} = 4$ .

- (i) Write down the set of differential equations for this problem. Are the conditions fulfilled?
- (ii) Plot the evolution of the degree distribution as a function of the normalized time for  $\tau = t/n \in [0, d_{\max}/2]$ .

HINT: In general one cannot expect to solve the system of differential equations analytically. But it is typically easy to solve them numerically. Here is how you do it in Mathematica. The following lines set up the differential equation we discussed in class and plots the solution.

```

cnds = {n[0] == 1}; (* initial conditions *)
eqns = {n'[u] == -2 n[u]^2}; (* set of diff equations *)
eqnspluscnds = Flatten[Join[eqns, cnds]]; (* put the two together *)
umax=10; (* solve equation up to this point *)
sol=Flatten[NDSolve[eqnspluscnds, {n}, {u, 0, umax}]]; (* solve the diff equation *)
Plot[Evaluate[{n[u]} /. sol], {u, 0.0, umax}] (* plot the solution *)

```

If you have more than one variable then it is often convenient to call them

$d[0][u]$ ,  $d[1][u]$ ,  $d[2][u]$ , ...

In this case you might have something like

```

cnds = {d[0][0] == ..., d[1][0]==..., ...};
eqns = {d[0]'[u] == ..., d[1]'[u]==..., ...};
eqnspluscnds = Flatten[Join[eqns, cnds]];
umax=...;
sol =
Flatten[NDSolve[eqnspluscnds, {d[0], d[1], ...}, {u, 0, umax}]];
Plot[Evaluate[{d[0][u], d[1][u], ...} /. sol], {u, 0.0, umax}]

```