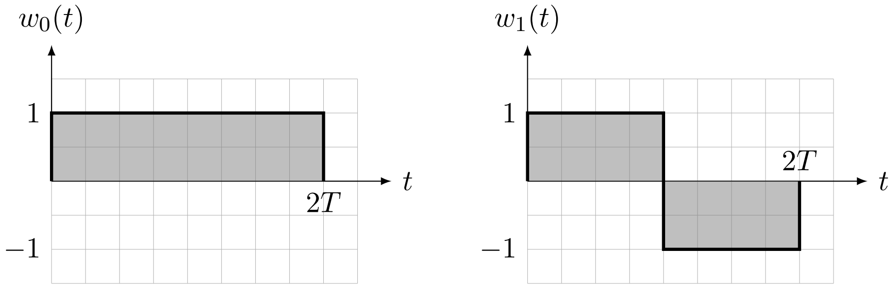
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 16 | Principles of Digital Communications

Problem Set 8 | Apr. 20, 2016

PROBLEM 1. Consider the signals $w_0(t)$ and $w_1(t)$ used to communicate 1 bit across the AWGN channel of power spectral density $\frac{N_0}{2}$.



(a) Determine an orthonormal basis $\{\psi_0(t), \psi_1(t)\}$ for the space spanned by $\{w_0(t), w_1(t)\}$ and find the corresponding codewords c_0 and c_1. Work out two solutions, one obtained via Gram–Schmidt and one in which $\psi_1(t)$ is a delayed version of $\psi_0(t)$. Which of the two solutions would you choose if you had to implement the system?

(b) Let X be a uniformly distributed binary random variable that takes values in $\{0, 1\}$. We want to communicate the value of X over an additive white Gaussian noise channel. When $X = 0$, we send $w_0(t)$, and when $X = 1$, we send $w_1(t)$. Draw the block diagram of an ML receiver based on a single matched filter.

(c) Determine the error probability P_e of your receiver as a function of T and N_0.

(d) Find a suitable waveform $v(t)$ such that the signals $\tilde{w}_0(t) = w_0(t) - v(t)$ and $\tilde{w}_1(t) = w_1(t) - v(t)$ have minimum energy. Plot the resulting waveforms.

(e) What is the name of the signaling scheme that uses signals such as $\tilde{w}_0(t)$ and $\tilde{w}_1(t)$? Argue that one obtains this kind of signaling scheme independently of the initial choice of $w_0(t)$ and $w_1(t)$.

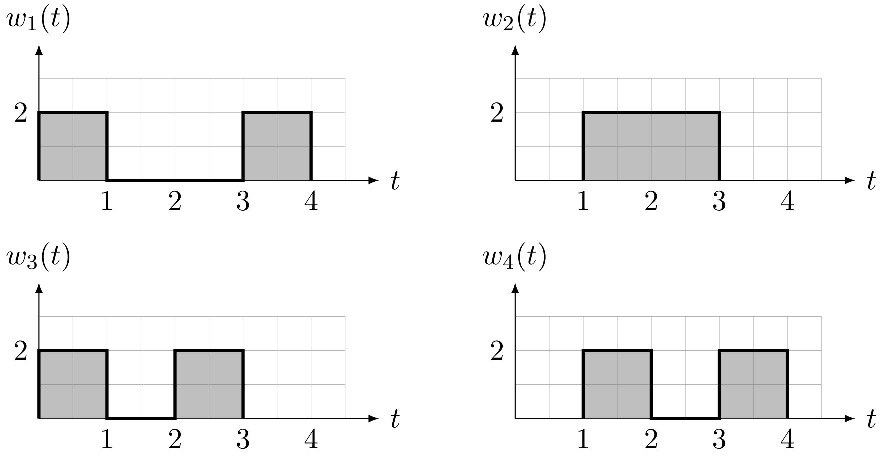
PROBLEM 2. Consider a set $\mathcal{W} = \{w_0(t), \dots, w_{m-1}(t)\}$ of mutually orthogonal signals with squared norm $\mathcal{E}$, each used with equal probability.

(a) Find the minimum-energy signal set $\tilde{\mathcal{W}} = \{\tilde{w}_0(t), \dots, \tilde{w}_{m-1}(t)\}$ obtained by translating the original set.

(b) Let $\tilde{\mathcal{E}}$ be the average energy of a signal picked at random within $\tilde{\mathcal{W}}$. Determine $\tilde{\mathcal{E}}$ and the energy saving $\mathcal{E} - \tilde{\mathcal{E}}$.

(c) Determine the dimension of the inner product space spanned by $\tilde{\mathcal{W}}$.

PROBLEM 3. Consider the signal set shown below. Each signal is equally likely to be chosen for transmission over an AWGN channel with power spectral density $\frac{N_0}{2}$.



(a) Represent the signal set using the four basis signals given by $\psi_1(t) = \psi(t)$, $\psi_2(t) = \psi(t-1)$, $\psi_3(t) = \psi(t-2)$, $\psi_4(t) = \psi(t-3)$, where

$$\psi(t) = \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(b) Use the union bound to find an upper bound to the error probability for the optimal receiver.

(c) Transform the four signals by a translation in order to obtain a minimum energy signal set. Sketch the new signal set $\{\tilde{w}_1(t), \tilde{w}_2(t), \tilde{w}_3(t), \tilde{w}_4(t)\}$.

(d) Use the Gram–Schmidt procedure to find an orthogonal basis for $\{\tilde{w}_1(t), \tilde{w}_2(t), \tilde{w}_3(t), \tilde{w}_4(t)\}$.

(e) Find the exact error probability of an optimal receiver designed for $\{\tilde{w}_1(t), \tilde{w}_2(t), \tilde{w}_3(t), \tilde{w}_4(t)\}$.

(f) Based on your answer to (e), what can you say about the error probability of the receiver in (b)?