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Exercice Set 7 : 14 April 2016  
Calcul Quantique

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**Exercice 1** *Effet des imperfections sur l'algorithme de Simon*

1. L'état juste après les deux premières portes est :

$$\begin{aligned}
 H_0 \otimes H_1 (|0\rangle \otimes |0\rangle \otimes |0\rangle) &= H_0 |0\rangle \otimes H_1 |0\rangle \otimes |0\rangle \\
 &= \frac{1}{2} (|0\rangle + (-1)^0 e^{i\varphi_0} |1\rangle) \otimes (|0\rangle + (-1)^0 e^{i\varphi_1} |1\rangle) \otimes |0\rangle \\
 &= \frac{1}{2} (|00\rangle + e^{i\varphi_1} |01\rangle + e^{i\varphi_0} |10\rangle + e^{i\varphi_0+i\varphi_1} |11\rangle) \otimes |0\rangle
 \end{aligned}$$

2. Après  $U_f$  :

$$\begin{aligned}
 &\frac{1}{2} \{U_f(|00\rangle \otimes |0\rangle) + e^{i\varphi_1} U_f(|01\rangle \otimes |0\rangle) + e^{i\varphi_0} U_f(|10\rangle \otimes |0\rangle) + e^{i\varphi_0+i\varphi_1} U_f(|11\rangle \otimes |0\rangle)\} \\
 &= \frac{1}{2} \{|00\rangle \otimes |0\rangle + e^{i\varphi_1} |01\rangle \otimes |0\rangle + e^{i\varphi_0} |10\rangle \otimes |1\rangle + e^{i\varphi_0+i\varphi_1} |11\rangle \otimes |0\rangle\},
 \end{aligned}$$

où on a utilisé  $f(00) = f(01) = 0$  et  $f(10) = f(11) = 1$ .

Enfin appliquons les deux dernières portes de Hadamard  $H \otimes H$  aux deux premiers qu-bits :

$$\begin{aligned}
 &\frac{1}{2} (H \otimes H |00\rangle) \otimes |0\rangle + \frac{1}{2} e^{i\varphi_1} (H \otimes H |01\rangle) \otimes |0\rangle + \\
 &\quad + \frac{1}{2} e^{i\varphi_0} (H \otimes H |10\rangle) \otimes |1\rangle + \frac{1}{2} e^{i(\varphi_0+\varphi_1)} (H \otimes H |11\rangle) \otimes |1\rangle \\
 &= \frac{1}{4} (|00\rangle + |01\rangle + |10\rangle + |11\rangle) \otimes |0\rangle \\
 &\quad + \frac{1}{4} e^{i\varphi_1} (|00\rangle - |01\rangle + |10\rangle - |11\rangle) \otimes |0\rangle \\
 &\quad + \frac{1}{4} e^{i\varphi_0} (|00\rangle + |01\rangle - |10\rangle - |11\rangle) \otimes |1\rangle \\
 &\quad + \frac{1}{4} e^{i(\varphi_0+\varphi_1)} (|00\rangle - |01\rangle - |10\rangle + |11\rangle) \otimes |1\rangle.
 \end{aligned}$$

L' état juste avant la mesure est :

$$\begin{aligned}
 &\frac{1}{4} \{(1 + e^{i\phi_1}) |00\rangle + (1 - e^{i\phi_1}) |01\rangle + (1 + e^{i\phi_1}) |10\rangle + (1 - e^{i\phi_1}) |11\rangle\} \otimes |0\rangle \\
 &+ \frac{e^{i\varphi_0}}{4} \{(1 + e^{i\phi_1}) |00\rangle + (1 - e^{i\phi_1}) |01\rangle + (1 + e^{i\phi_1}) |10\rangle + (1 - e^{i\phi_1}) |11\rangle\} \otimes |1\rangle
 \end{aligned}$$

3. Mesure des deux premiers qu-bits :

$$\begin{aligned}\text{Prob}[00] &= \frac{1}{16} |1 + e^{i\varphi_1}|^2 + \frac{1}{16} |1 + e^{i\varphi_1}|^2 = \frac{1}{2} \cos^2 \frac{\varphi_1}{2}, \\ \text{Prob}[01] &= \frac{1}{16} |1 - e^{i\varphi_1}|^2 + \frac{1}{16} |1 - e^{i\varphi_1}|^2 = \frac{1}{2} \sin^2 \frac{\varphi_1}{2}, \\ \text{Prob}[10] &= \frac{1}{16} |1 + e^{i\varphi_1}|^2 + \frac{1}{16} |1 + e^{i\varphi_1}|^2 = \frac{1}{2} \cos^2 \frac{\varphi_1}{2}, \\ \text{Prob}[11] &= \frac{1}{16} |1 - e^{i\varphi_1}|^2 + \frac{1}{16} |1 - e^{i\varphi_1}|^2 = \frac{1}{2} \sin^2 \frac{\varphi_1}{2}.\end{aligned}$$

4. Pour toujours trouver un vecteur dans  $H^\perp = \{(0, 0); (1, 0)\}$ , il est nécessaire et suffisant de prendre  $\varphi_1 = 0$  et  $\varphi_0$  quelconque :

$$\begin{cases} \text{Prob}[00] = \frac{1}{2} \\ \text{Prob}[10] = \frac{1}{2} \end{cases} \implies \text{Prob}[(0, 0) \text{ ou } (1, 0)] = 1,$$

$$\begin{cases} \text{Prob}[01] = 0 \\ \text{Prob}[11] = 0 \end{cases} \implies \text{Prob}[(0, 1) \text{ ou } (1, 1)] = 0.$$

En général, avec  $\varphi_1 \neq 0$ , on a

$$\begin{aligned}\text{Prob}[00 \text{ ou } 10] &= \frac{1}{2} \cos^2 \frac{\varphi_1}{2} + \frac{1}{2} \cos^2 \frac{\varphi_1}{2} = \cos^2 \frac{\varphi_1}{2}, \\ \text{Prob}[01 \text{ ou } 11] &= \frac{1}{2} \sin^2 \frac{\varphi_1}{2} + \frac{1}{2} \sin^2 \frac{\varphi_1}{2} = \sin^2 \frac{\varphi_1}{2}.\end{aligned}$$