Exercise Set 6 : 7 April 2016
Calcul Quantique

Exercise 1 *Un petit algorithme quantique*

a)

$$H|0\rangle \otimes |u\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |u\rangle + \frac{1}{\sqrt{2}}|1\rangle \otimes |u\rangle$$

$$CUH|0\rangle \otimes |u\rangle = \frac{1}{\sqrt{2}}|0\rangle \otimes |u\rangle + \frac{1}{\sqrt{2}}e^{2\pi i\varphi}|1\rangle \otimes |u\rangle$$

$$\begin{aligned} HCUH|0\rangle \otimes |u\rangle &= \frac{1}{2}(|0\rangle + |1\rangle) \otimes |u\rangle + \frac{e^{2\pi i\varphi}}{2}(|0\rangle - |1\rangle) \otimes |u\rangle \\ &= \frac{1+e^{2\pi i\varphi}}{2}|0\rangle \otimes |u\rangle + \frac{1-e^{2\pi i\varphi}}{2}|1\rangle \otimes |u\rangle \\ &= e^{\pi i\varphi}(\cos\pi\varphi|0\rangle \otimes |u\rangle - i\sin\pi\varphi|1\rangle \otimes |u\rangle) \end{aligned}$$

b)

$$\text{Prob}(0) = \cos^2\pi\varphi \quad \text{et} \quad \text{Prob}(1) = \sin^2\pi\varphi$$

c) Si on applique U^k au lieu de U on trouve la sortie :

$$e^{i\pi k\varphi}(\cos(\pi k\varphi)|0\rangle \otimes |u\rangle - i\sin(\pi k\varphi)|1\rangle \otimes |u\rangle)$$

Si $\varphi = \frac{\varphi_1}{2} + \frac{\varphi_2}{2^2} + ... + \frac{\varphi_{t-1}}{2^{t-1}} + \frac{\varphi_t}{2^t}$ en prenant $k = 2^{t-1}$ on observe 0 avec probabilité

$$\text{Prob}(0) = \cos^2(\pi\varphi_{t-1} + \frac{\pi\varphi_t}{2}) = \cos^2(\frac{\pi\varphi_t}{2}) = \begin{cases} 1 & \text{si } \varphi_t = 0 \\ 0 & \text{si } \varphi_t = 1 \end{cases}$$