ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE
School of Computer and Communication Sciences

**Handout 5** Statistical Physics for Communication and Computer Science
Homework 5 March 25, 2015

**Problem 1** (Min-Sum Mesage Passing rules)**.** In class we discussed how to compute the marginal of a multivariate function $f(x_1, \dots, x_n)$ efficiently, assuming that the function can be factorized into factors involving only few variables and that the corresponding factor graph is a tree. We accomplished this by formulating a message-passing algorithm. The messages are functions over the underlying alphabet. Functions are passed on edges. The algorithm starts at the leaf nodes and we discussed how messages are computed at variable and at function nodes.

Recall from the derivation that the main property we used was the *distributive law*. Consider now the following generalization. Consider the so-called *commutative semiring* of extended real numbers (including $\infty$) with the two operations min and $+$ (instead of the usual operations $+$ and $*$).

(i) Show that both operations are commutative.

(ii) Show that the identity element under min is $\infty$ and that the identity element under $+$ is 0.

(iii) Show that the distributive law holds.

(iv) If we formally exchange in our original marginalization $+$ with min and $*$ with $+$, what corresponds to the marginalization of a function?

(v) What are the message passing rules and what is the initialization?

More generally, there is a message passing algorithm for any commutative semi-ring.

**Problem 2** (Application to the Lasso estimate)**.** The goal of this problem is to show that in case the factor graph associated to the measurement matrix is a tree we can solve the Lasso minimization problem by using the min-sum algorithm. Recall that the Lasso estimate is

$$\hat{\underline{x}}^{\text{lasso}}(\underline{y}) = \text{argmin}_{\underline{x}} \left\{ \frac{1}{2} \|\underline{y} - A\underline{x}\|_2^2 - \lambda \|x\|_1 \right\}. \tag{1}$$

Consider first the minimum cost given that $x_i$ is fixed.

$$C(x_i) = \min_{\sim x_i} \left\{ \frac{1}{2} \|\underline{y} - A\underline{x}\|_2^2 - \lambda \|x\|_1 \right\}. \tag{2}$$

where $\min_{\sim x_i}$ denotes minimization of the expression in the bracket with respect to all variables, except $x_i$ which is held fixed. $C(x_i)$ is a function of a single real variable whose minimizer yields the $i$-th component of $\hat{\underline{x}}^{\text{lasso}}(\underline{y})$.

Consider the Tanner graph in figure 6.7 in the notes and write down the factors associated to factor nodes. Pick your favourite variable, say variable 4, and describe the steps of the min-sum algorithm for the computation of $C(x_4)$.