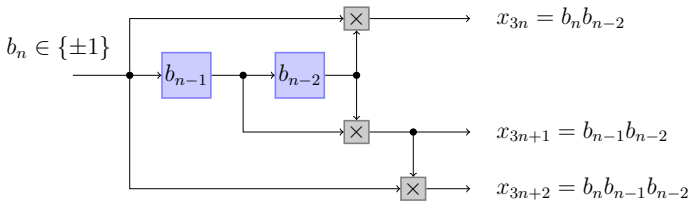
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 25 Principles of Digital Communications
Problem Set 11 May 13, 2015

PROBLEM 1. *(Rate 1/3 Convolutional Code)* For the convolutional encoder depicted below:



(a) Draw the state diagram and the detour flow graph.

(b) Suppose that the serialized encoder output symbols are scaled so that the resulting energy per bit is $\mathcal{E}_b$ and are sent over the discrete-time AWGN channel of noise variance $\sigma^2 = N_0/2$. Derive an upper bound to the bit error probability assuming that the decoder implements the Viterbi algorithm.

PROBLEM 2. *(Rate 2/3 Convolutional Code)* The following equations describe the output of a convolutional encoder that in each epoch takes $k_0 = 2$ input symbols from $\{\pm 1\}$ and outputs $n_0 = 3$ symbols from the same alphabet:

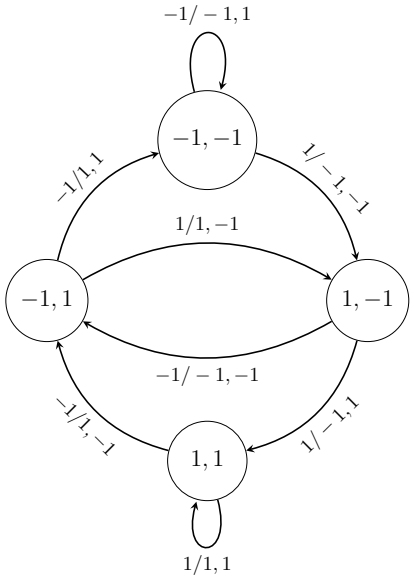
$$\begin{aligned} x_{3n} &= b_{2n} b_{2n-1} b_{2n-2}, \\ x_{3n+1} &= b_{2n+1} b_{2n-2}, \\ x_{3n+2} &= b_{2n+1} b_{2n} b_{2n-2}. \end{aligned}$$

(a) Draw an implementation of the encoder based on delay elements and multipliers.

(b) Draw the state diagram.

(c) Suppose that the serialized encoder output symbols are scaled so that the resulting energy per bit is $\mathcal{E}_b$ and are sent over the discrete-time AWGN channel of noise variance $\sigma^2 = N_0/2$. Derive an upper bound to the bit error probability assuming that the decoder implements the Viterbi algorithm.

PROBLEM 3. *(Convolutional Encoder, Decoder and Error Probability)* Consider the convolutional code described by the following state diagram:



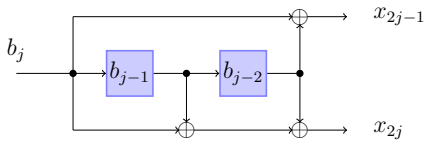
(a) Draw the encoder.

(b) As a function of the energy per bit $\mathcal{E}_b$, upper bound the bit error probability of the Viterbi algorithm when the scaled encoder output sequence is transmitted over the discrete-time AWGN channel of noise variance $\sigma^2 = N_0/2$.

PROBLEM 4. *(Viterbi for the Binary Erasure Channel)* Consider the convolutional encoder depicted below with inputs and outputs over $\{0, 1\}$ and addition modulo 2. Its output is sent over the binary erasure channel described by

$$\begin{aligned} P_{Y|X}(0|0) &= P_{Y|X}(1|1) = 1 - \epsilon, \\ P_{Y|X}(?|0) &= P_{Y|X}(?|1) = \epsilon, \\ P_{Y|X}(1|0) &= P_{Y|X}(0|1) = 0, \end{aligned}$$

where $0 < \epsilon < 0.5$.



(a) Draw a trellis section that describes the encoder map.

(b) Derive the branch metric and specify whether a maximum likelihood decoder chooses the path with largest or smallest path metric.

(c) Suppose that the initial encoder state is $(0, 0)$ and that the channel output is $\{0, ?, ?, 1, 0, 1\}$. What is the most likely information sequence?

(d) Derive an upper bound to the bit error probability.