ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Exercise 8 **Graph Theory Applications**

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Problem 1. You have been given the task of preparing question paper in computer science with 100 questions. The questions can come from 10 different categories, 10 for each category. To assist you, there is a question bank of 500 hundred questions, but each question has several categories attached to it. Formulate a flow problem to check whether it is possible to get all the questions you want from the question bank.

Problem 2. Show how to transform the following types of flow problems to a standard flow problem.

1. Each vertex in the network has a capacity and the flow passing through the vertex cannot exceed this number.
2. Each edge has a lower bound on the flow that passes through it. Note that edge capacities are upper bounds on the flow.

Problem 3. In a public building such as a movie theater, it is important to have a plan of exit in the event of a fire. We will design such an emergency exit plan in this question using max-ows. Suppose a movie theater is represented by a graph $G = (V; E)$, where each room, landing, or other location is represented by a vertex and each corridor or stairway is represented by an edge. Each corridor has the same capacity c, meaning that at most c people can pass through the corridor at once. Traversing a corridor from one end to the other takes one timestep. (Traversing a room takes zero time.)

1. Suppose all people are initially in a single room s, and there is a single exit t. Show how to use maximum ow to find a fastest way to get everyone out of the building. (Hint: create another graph G_0 that has vertices to represent each room at each time step.)
2. Show how the same idea can be used when people are initially in multiple locations and there are multiple exits.
3. Finally, suppose that it takes different (but integer) amounts of time to cross different corridors or stairways, and that for each such corridor or stairway e, you are also given an integer $t(e)$ which is the number of seconds required to cross e. Now show how to transform your algorithm in Part (1) to find a fastest way to get everyone out of the building.