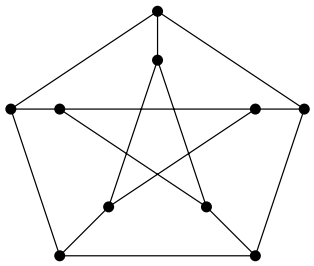
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

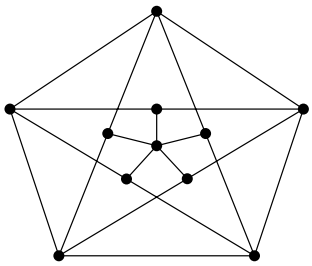
Exercise 8 **Graph Theory Applications**

Date: May 2, 2013 Spring 2013

Problem 1. Determine the chromatic number of the following graphs. (Hint: Use Brook's theorem.)



(a) Petersen Graph

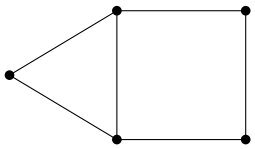


(b) Grötzsch Graph

Problem 2. Prove that any connected graph $G = (V, E)$ can be properly vertex-colored using atmost $|E| - |V| + 3$ colors.

Problem 3. On a plane there are a few lines such that no three lines have a common intersection. Consider the graph G whose vertex set is the set of intersection points and line sections between them represent edges (i.e. two vertices are adjacent iff they are neighboring intersection points on one of the lines). Show that $\chi(G) \leq 3$. (You can assume that no two intersection points have the same x coordinates.)

Problem 4. Calculate the chromatic polynomial of the following graph.



Problem 5. Prove that the chromatic polynomial of any tree with n vertices is $k(k-1)^{n-1}$. (Hint: Use induction.)

Problem 6. Use the above result and induction to show that the chromatic polynomial of the n-cycle is $(k-1)^n + (-1)^n(k-1)$ where $n \geq 3$.