Exercise 6   **Graph Theory Applications**
Date: April 18, 2013   Spring 2013

Problem 1. What is χ' for the following graphs?

(a) Path Graph P_n (b) Cycle Graph C_n (c) Wheel Graph W_{n+1}
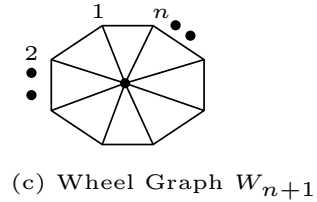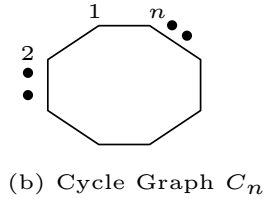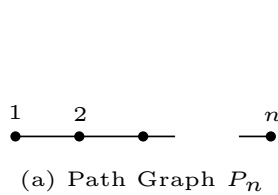
Problem 2. Show that for a nonempty simple regular graph with odd number of vertices $\chi' = \Delta+1$.

Problem 3. If m^* is the size of the maximum matching of a graph with m edges prove that

$$\chi' \geq \lceil \frac{m}{m^*} \rceil$$

Problem 4. Show, by finding an appropriate edge colouring, that $\chi'(K_{m,n}) = \Delta(K_{m,n})$.

Problem 5. A graph $G(V, E)$ is uniquely k-edge colourable if any two proper k-edge colourings of G induce the same partition of E. Show that every uniquely 3-edge-colourable 3-regular graph is Hamiltonian.

Problem 6. Let G be a 3-regular graph with $\chi' = 4$. Prove that G is not Hamiltonian.