ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Exercise 5 **Graph Theory Applications**

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Problem 1. What is the length of the maximum matching in the cycle graph on n vertices? Can you give a closed form expression?

Problem 2. For $n \geq 1$, consider a graph $G_n = (V_n, E_n)$ constructed as follows.
$V_n =$ Set of all the binary n-tuples of the form of $(b_1, b_2, \cdots, b_n)$ where $b_i \in \{0, 1\}$.
$E_n =$ We connect any two vertices in V_n iff their Hamming distance is exactly 1 i.e. the bit patterns differ only at one position. Prove that G_n has a perfect matching.

Problem 3. Show that a tree cannot have two distinct perfect matchings. (Two matchings are distinct if there exists an edge that is contained in one matching but not the other.)

Problem 4. Two people play a game on a graph G by alternately selecting distinct vertices $v_1, v_2, v_3, \ldots$ such that for $i > 0$, v_i is adjacent to v_{i-1}. The last player who is able to select a vertex wins. If player 1 is the first to choose a vertex, show that G has a perfect matching if and only if there is a winning strategy for player 2.