ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Problem 1. Suppose that n people are attending a party and there are some handshakes between different people in the party. Show that there are at least two persons who have shaken hands with the same number of people.

Problem 2. Balls of 8 different colors are placed in 6 jars. There are 20 balls of each color. Use the pigeonhole principle to show that there must be a jar containing two pairs from two different colors of balls (for example, there is a jar containing at least two blue and at least two green balls).

Problem 3. (a) Is there a graph with degrees 1, 2, 2, 2, 3, 3, 4, 4, 4, 5, 5, 7, 7 ?

(b) Is there a bipartite graph with degrees 3, 3, 3, 3, 3, 5, 6, 6, 6, 6, 6, 6 ?

(c) Is there a simple graph (i.e., with no multiple edges or self loops) with degrees 1, 1, 4, 4, 4, 5, 6, 7, 8 ?

(d) Is there a forest with degrees 1, 1, 2, 3, 3 ?

Problem 4. If the maximum degree of a simple, connected bipartite graph G is $\Delta(G)$, prove that the maximum number of vertices in it is

$$|V(G)| \leq 2\frac{(\Delta(G) - 1)^{\mathrm{diam(G)}} - 1}{\Delta(G) - 2}$$

Problem 5. Show that for every connected undirected graph G it holds that $\mathrm{rad}(G) \leq \mathrm{diam}(G) \leq 2\mathrm{rad}(G)$. Find a graph where $\mathrm{rad}(G) = \mathrm{diam}(G)$, and a graph where $2\mathrm{rad}(G) = \mathrm{diam}(G)$.