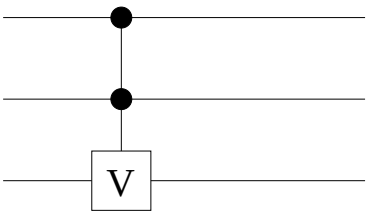
Quantum Information and Computation
9 December 2011

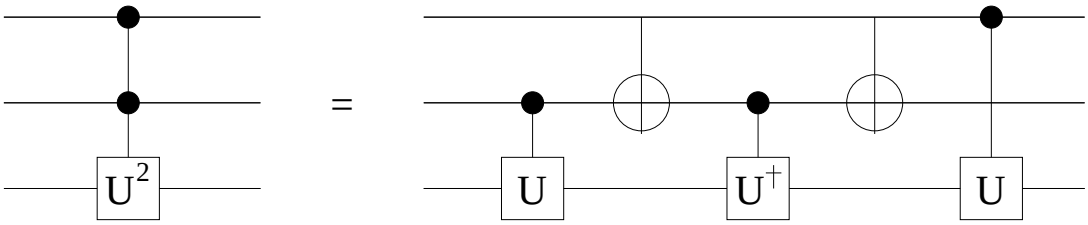
Homework 9
Quantum Information and Computation

Exercice 1

Let V a 2×2 unitary matrix. The "double control-V" gate denoted CCV is defined by the circuit



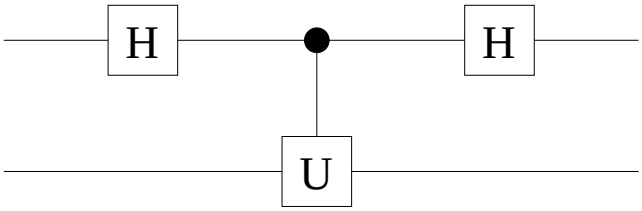
1a) Show that for all 2×2 unitary U :



1b) Find U that realizes the quantum Toffoli gate CCNOT ? Give the explicit matrix U.

Exercice 2

Let U a unitary matrix and $|u\rangle$ an eigen-vector : $U|u\rangle = \exp(2\pi i\varphi)|u\rangle$. Consider the circuit :



3a) Calculate the poutput for the initial state $|0\rangle \otimes |u\rangle$.

3b) Calculate the probability to observe the first bit in the state $|0\rangle$ (at the output). Same question for the probability to observe it in the state $|1\rangle$. Same question for the probabilities to observe $\frac{|0\rangle+|1\rangle}{\sqrt{2}}$; $\frac{|0\rangle-|1\rangle}{\sqrt{2}}$; $\frac{|0\rangle+i|1\rangle}{\sqrt{2}}$ et $\frac{|0\rangle-i|1\rangle}{\sqrt{2}}$ at the output.

3c) Suppose we replace U by U^k, k integer, in the circuit above. Let $\varphi = 0, \varphi_1\varphi_2...\varphi_t$ the binary expansion of $0 < \varphi < 1$. How does one have to choose k in order to determine the least significant bit φ_t with just one measurement ?