ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Principles of Digital Communications: Assignment date: May 9, 2012
Summer Semester 2012 Due date: May 16, 2012

Homework 11

Reading Part for next Wednesday: From Section 6.4 (Bit-Error Probability) till end of chapter 6 (9p).

Problem 1. *(Project Plan)*

Now where you have implemented the transmitter, it is time to implement the receiver. By next Wednesday try to do the following two tasks: 1) Implement the receiver portion of the system. 2) By putting different parts together, try to get a live system that is able to send some amount of information and receive it. At this stage, your focus should be on getting a simpleset version of your system to run and you should not be concerned about the quality (error probability, delay, speed, etc) of it. Next Wednesday please bring your laptop so that we can have a look at a small demo of your syestem.

Problem 2. *(Power Spectral Density)*
Block-Orthogonal signaling may be the simplest coding method that achieves $Pr\{e\} \to 0$ as $N \to \infty$ for a non-zero data rate. However, we have seen in class that the price to pay is that block-orthogonal signaling requires infinite bandwidth to make $Pr\{e\} \to 0$. This may be a small problem for one space explorer communicating to another; however, for terrestrial applications, there are always constraints on the bandwidth consumption. Therefore, in the examination of any coding method, an important issue is to compute its bandwidth consumption. Compute the bandwidth occupied by the rate $\frac{1}{2}$ convolutional code studied in this chapter. The signal that is put onto the channel is given by

$$X(t) = \sum_{i=-\infty}^{\infty} X_i \sqrt{E_s}\psi(t - iT_s), \tag{1}$$

where $\psi(t)$ is some unit-energy function of duration T_s and we assume that the trellis extends to infinity on both ends, but as usual we actually assume that the signal is the wide-sense stationary signal

$$\tilde{X}(t) = \sum_{i=-\infty}^{\infty} X_i \sqrt{E_s}\psi(t - iT_s - T_0), \tag{2}$$

where T_0 is a random delay which is uniformly distributed over the interval $[0, T_s)$.

1. Find the expectation $E[X_i X_j]$ for $i = j$, for $(i, j) = (2n, 2n+1)$ and for $(i, j) = (2n, 2n+2)$ for the convolutional code that was studied in class and give the autocorrelation function $R_X[i - j] = E[X_i X_j]$ for all i and j. *Hint:* Consider the infinite trellis of the code. Recall that the convolution code studied in the class can be defined as

$$\begin{aligned} X_{2n} &= D_n D_{n-2} \\ X_{2n+1} &= D_n D_{n-1} D_{n-2} \end{aligned}$$

2. Find the autocorrelation function of the signal $\tilde{X}(t)$, that is

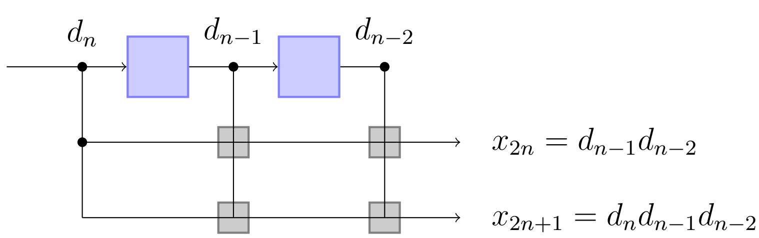
$$R_{\tilde{X}}(\tau) = E[\tilde{X}(t)\tilde{X}(t+\tau)] \tag{3}$$

   in terms of $R_X[k]$ and $R_\psi(\tau) = \frac{1}{T_s}\int_{-\infty}^{\infty} \psi(t+\tau)\psi(t)dt$.

3. Give the expression of power spectral density of the signal $\tilde{X}(t)$.

4. Find and plot the power spectral density that results when $\psi(t)$ is a rectangular pulse of width T_s centered at 0.

Problem 3. *(Trellis Section)*
Draw one section of the trellis for the convolutional encoder shown below, where it is assumed that d_n takes values in $\{\pm 1\}$. Each (valid) transition in the trellis should be labeled with the corresponding outputs (x_{2n}, x_{2n+1}).



Problem 4. *(Branch Metric)*
Consider the convolutional code described by the trellis section below on the left. At time n the output of the encoder is (x_{2n}, x_{2n+1}). The transmitted waveform is $\sqrt{E_s}\sum x_k \psi(t - kT)$ where $\psi(t)$ is a (unit energy) Nyquist pulse. At the receiver we perform matched filtering with the filter matched to $\psi(t)$ and sample the output of the match filter at kT. Suppose the output of the matched filter corresponding to (x_{2n}, x_{2n+1}) is $(y_{2n}, y_{2n+1}) = (1, -2)$. Find the branch metric values to be used by the Viterbi algorithm and enter them into the trellis section on the right.

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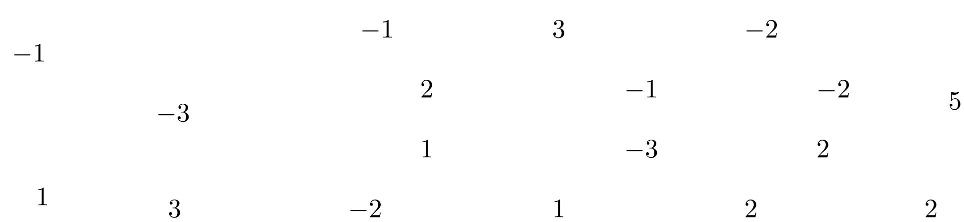

Problem 6. *(Intersymbol Interference)*

An information sequence $\underline{U} = (U_1, U_2, \ldots, U_5)$, $U_i \in \{0, 1\}$ is transmitted over a noisy intersymbol interference channel. The ith sample of the receiver-front-end filter (e.g. a filter matched to the pulse used by the sender)

$$Y_i = S_i + Z_i,$$

where the noise Z_i forms an independent and identically distributed (i.i.d.) sequence of Gaussian random variables,

$$S_i = \sum_{j=0}^{\infty} U_{i-j} h_j, \qquad i = 1, 2, \ldots$$

and

$$h_i = \begin{cases} 1, & i = 0 \\ -2, & i = 1 \\ 0, & \text{otherwise.} \end{cases}$$

You may assume that $U_i = 0$ for $i \geq 6$ and $i \leq 0$.

1. Rewrite S_i in a form that explicitly shows by which symbols of the information sequence it is affected.

2. Sketch a trellis representation of a finite state machine that produces the output sequence $\underline{S} = (S_1, S_2, \ldots, S_6)$ from the input sequence $\underline{U} = (U_1, U_2, \ldots, U_5)$. Label each trellis transition with the specific value of $U_i | S_i$.

3. Specify a metric $f(\underline{s}, \underline{y}) = \sum_{i=1}^{6} f(s_i, y_i)$ whose minimization or maximization with respect to $\underline{s}$ leads to a maximum likelihood decision on $\underline{S}$. Specify if your metric needs to be minimized or maximized. *Hint:* Think of a vector channel $\underline{Y} = \underline{S} + \underline{Z}$, where $\underline{Z} = (Z_1, \ldots, Z_6)$ is a sequence of i.i.d. components with $Z_i \sim \mathcal{N}(0, \sigma^2)$.

4. Assume $\underline{Y} = (Y_1, Y_2, \cdots, Y_5, Y_6) = (2, 0, -1, 1, 0, -1)$. Find the maximum likelihood estimate of the information sequence $\underline{U}$. *Please:* Do *not* write into the trellis that you have drawn in Part *(b)*; work on a copy of that trellis.