ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Principles of Digital Communications: Assignment date: May 2, 2012
Summer Semester 2012 Due date: May 9, 2012

Homework 10

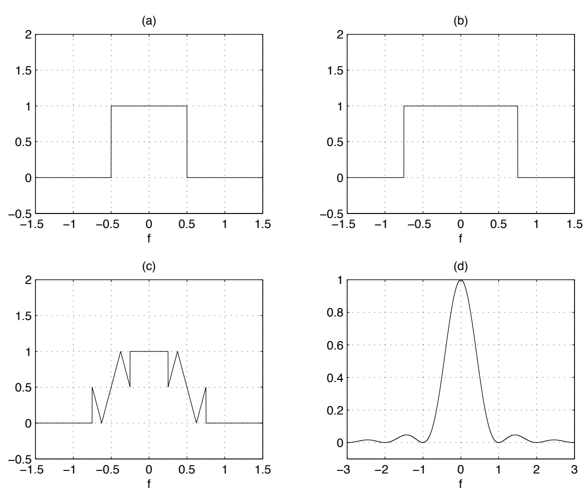
Reading Part for next Wednesday: From the beginning of Chapter 6 (Convolutional Coding and Viterbi Decoding) till the end of Section 6.3 (The Receiver).

Problem 1. *(Project Plan)*

Now where you have handed in a design of your system it is time to implement it. By next Wednesday implement the transmitter portion of the system. Make sure to have a flexible implementation which allows you to adjust all the major parameters in your system (time constants, number of bits you send, etc.). Next Wednesday please bring your laptop so that we can have a look at a small demo of your transmitter.

Problem 2. *(Nyquist Criterion)*

1. Consider the following $|\theta_{\mathcal{F}}(f)|^2$. The unit on the frequency axis is $\frac{1}{T}$ and the unit on the vertical axis is T. Which ones correspond to Nyquist pulses $\theta(t)$ for symbol rate $\frac{1}{T}$?(*Note:* Figure *(d)* shows a sinc^2 function.)



2. Design a (non-trivial) Nyquist pulse yourself.

3. Sketch the block diagram of a binary communication system that employs Nyquist pulses. Write out the formula for the signal after the matched filter. Explain the advantages of using Nyquist pulses.

Problem 3. *(Mixed Questions)*

1. Consider the signal $x(t) = \cos(2\pi t)\left(\frac{\sin(\pi t)}{\pi t}\right)^2$. Assume that we sample $x(t)$ with sampling period T. What is the maximum T that guarantees signal recovery?

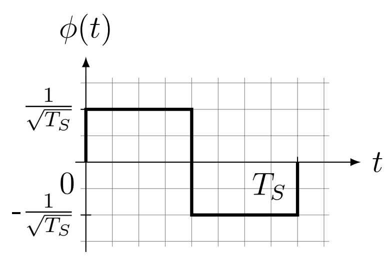
2. Consider the three signals $s_1(t) = 1, s_2(t) = \cos(2\pi t), s_3(t) = \sin^2(\pi t)$, for $0 \le t \le 1$. What is the dimension of the signal space spanned by $\{s_1(t), s_2(t), s_3(t)\}$?

3. You are given a pulse $p(t)$ with spectrum $p_{\mathcal{F}}(f) = \sqrt{T(1 - |f|T)}$, $0 \le |f| \le \frac{1}{T}$. What is the value of $\int p(t)p(t-3T)dt$?

Problem 4. *(Power Spectrum: Manchester Pulse)*
In this problem you will derive the power spectrum of a signal

$$X(t) = \sum_{i=-\infty}^{\infty} X_i \phi(t - iT_s - \Theta)$$

where $\{X_i\}_{i=-\infty}^{\infty}$ is an iid sequence of uniformly distributed random variables taking values in $\{\pm\sqrt{E_s}\}$, Θ is uniformly distributed in the interval $[0, T_s]$, and $\phi(t)$ is the so-called Manchester pulse shown in the following figure



1. Let $r(t) = \sqrt{\frac{1}{T_s}} 1_{[-\frac{T_s}{4}, \frac{T_s}{4}]}(t)$ be a rectangular pulse. Plot $r(t)$ and $r_{\mathcal{F}}(f)$, both appropriately labeled, and write down a mathematical expression for $r_{\mathcal{F}}(f)$.

2. Derive an expression for $|\phi_{\mathcal{F}}(f)|^2$. Your expression should be of the form $A\frac{\sin^m()}{()^n}$ for some A, m, and n. Hint: Write $\phi(t)$ in terms of $r(t)$ and recall that $\sin x = \frac{e^{jx} - e^{-jx}}{2j}$ where $j = \sqrt{-1}$.

3. Determine $R_X[k] \triangleq E[X_{i+k}X_i]$ and the power spectrum

$$S_X(f) = \frac{|\phi_{\mathcal{F}}(f)|^2}{T_S} \sum_{k=-\infty}^{\infty} R_X[k] e^{-j2\pi k f T_s}.$$