

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 5
Homework 3

Information Theory and Coding
Oct. 4, 2011

PROBLEM 1. (Problem 2.1 from textbook)

Coin flips. A fair coin is flipped until the first head occurs. Let X be the number of flips required.

- (a) Find the entropy $H(X)$ in bits. The following expressions may be useful:

$$\sum_{n=1}^{\infty} r^n = \frac{r}{1-r}, \quad \sum_{n=1}^{\infty} nr^n = \frac{r}{(1-r)^2}.$$

- (b) A random variable X is drawn according to this distribution. Find an “efficient” sequence of yes-no questions to determine X of the form “Is X contained in the set S ?” Compare $H(X)$ to the expected number of questions to determine X .

PROBLEM 2. (Problem 2.23 from textbook)

Run length coding. Let X_1, X_2, \dots, X_n be (possibly dependent) binary random variables. Suppose one calculates the run lengths $\mathbf{R} = (R_1, R_2, \dots)$ of this sequence (in the order they occur). For example, the sequence $\mathbf{x} = 0001100100$ yields the run lengths $\mathbf{R} = (3, 2, 2, 1, 2)$. Compare $H(X_1, X_2, \dots, X_n)$, $H(\mathbf{R})$ and $H(X_n, \mathbf{R})$. Show all equalities and inequalities and bound all the differences.

PROBLEM 3. Let $p_{XY}(x, y)$ be given by

$X \backslash Y$	0	1
0	1/3	1/3
1	0	1/3

Find

- (a) $H(X), H(Y)$.
- (b) $H(X|Y), H(Y|X)$.
- (c) $H(X, Y)$.
- (d) $H(Y) - H(Y|X)$.
- (e) $I(X; Y)$.
- (f) Draw a Venn diagram for the quantities in (a) through (e).

