ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

School of Computer and Communication Sciences

Principles of Digital Communications: Spring Semester 2012

Assignment date: Apr 18, 2011
Due date: Apr 25, 2011

Homework 8

Reading Part for this week: Chapter 4 until and including Section 4.4.2 (Growing BT Linearly with k).

Reading Part for next Wednesday: From Section 4.4.3 (Growing BT Exponentially with k) till Section 5.2 (The Ideal Lowpass Case). Read also Appendix 5A (Fourier Series) (10p)

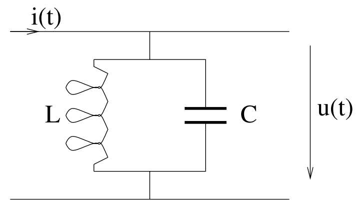
Problem 1. *(Matched Filter Implementation)*

In this problem, we consider the implementation of matched filter receivers. In particular, we consider Frequency Shift Keying (FSK) with the following signals:

$$s_j(t) = \begin{cases} \sqrt{\frac{2}{T}} \cos 2\pi \frac{n_j}{T} t, & \text{for } 0 \le t \le T, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

where $n_j \in \mathbb{Z}$ and $0 \le j \le m-1$. Thus, the communications scheme consists of m signals $s_j(t)$ of different frequencies $\frac{n_j}{T}$

1. Determine the impulse response $h_j(t)$ of the matched filter for the signal $s_j(t)$. Plot $h_j(t)$.
2. Sketch the matched filter receiver. How many matched filters are needed?
3. For $-T \le t \le 3T$, sketch the output of the matched filter with impulse response $h_j(t)$ when the input is $s_j(t)$. (*Hint: You may do a qualitative plot or use use Matlab.*)
4. Consider the following ideal resonance circuit:



For this circuit, the voltage response to the input current $i(t) = \delta(t)$ is

$$h(t) \quad = \quad \frac{1}{C} \cos \frac{t}{\sqrt{LC}}. \tag{2}$$

Show how this can be used to implement the matched filter for signal $s_j(t)$. Determine how L and C should be chosen. (*Hint:* Suppose that $i(t) = s_j(t)$. In that case, what is $u(t)$?)

Problem 2. *(On-Off Signaling)*
Consider the binary hypothesis testing problem specified by:

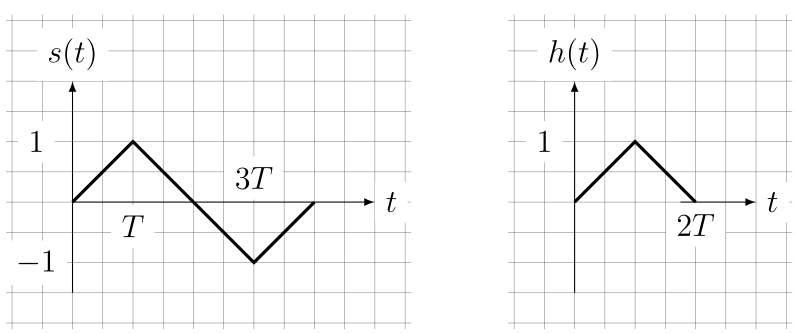
$$\begin{aligned} H = 0 &\; : \; Y(t) = s(t) + N(t) \\ H = 1 &\; : \; Y(t) = N(t) \end{aligned}$$

where $N(t)$ is AWGN (Additive White Gaussian Noise) of power spectral density $N_0/2$ and $s(t)$ is the signal shown in the left figure.

1. Describe the maximum likelihood receiver for the observable $Y(t)$, $t \in \mathbb{R}$.

2. Determine the error probability for the receiver you described in *(1)*.

3. Can you realize your receiver of part *(1)* using a filter with impulse response $h(t)$ shown in the right figure?



Problem 3. *(Receiver for Non-White Gaussian Noise)*
We consider the receiver design problem for signals used in non-white additive Gaussian noise. That is, we are given a set of signals $\{s_j(t)\}_{j=0}^{m-1}$ as usual, but the noise added to those

signals is no longer white; rather, it is a Gaussian stochastic process with a given power spectral density

$$S_N(f) \;=\; G^2(f), \tag{3}$$

where we assume that $G(f) \neq 0$ inside the bandwidth of the signal set $\{s_j(t)\}_{j=0}^{m-1}$. The problem is to design the receiver that minimizes the probability of error.

1. Find a way to transform the above problem into one that you can solve, and derive the optimum receiver.

2. Suppose there is an interval $[f_0, f_0+\Delta]$ inside the bandwidth of the signal set $\{s_j(t)\}_{j=0}^{m-1}$ for which $G(f) = 0$. What do you do? Describe in words.

Problem 4. *(QAM Receiver)*
Consider a transmitter which transmits waveforms of the form,

$$s(t) \;=\; \begin{cases} s_1\sqrt{\frac{2}{T}}\cos 2\pi f_c t + s_2\sqrt{\frac{2}{T}}\sin 2\pi f_c t, & \text{for } 0 \leq t \leq T, \\ 0, & \text{otherwise,} \end{cases} \tag{4}$$

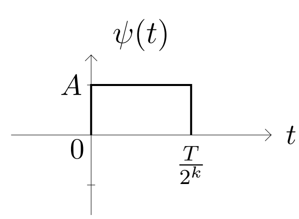
where $2f_cT \in \mathbb{Z}$ and $(s_1, s_2) \in \{(\sqrt{E}, \sqrt{E}), (-\sqrt{E}, \sqrt{E}), (-\sqrt{E}, -\sqrt{E}), (\sqrt{E}, -\sqrt{E})\}$ with equal probability. The signal received at the receiver is corrupted by AWGN of power spectral density $\frac{N_0}{2}$.

1. Specify the receiver for this transmission scheme.

2. Draw the decoding regions and find the probability of error.

Problem 5. *(Signaling Scheme Example)*
We communicate by means of a signal set that consists of 2^k waveforms, $k \in \mathbb{N}$. When the message is i we transmit signal $s_i(t)$ and the received observes $R(t) = s_i(t) + N(t)$, where $N(t)$ denotes white Gaussian noise with double-sided power spectral density $\frac{N_0}{2}$. Assume that the transmitter uses the position of a pulse $\psi(t)$ in an interval $[0, T]$, in order to convey the desired hypothesis, i.e., to send hypothesis $i \in \{0, 1, 2, \cdots, 2^k - 1\}$, the transmitter sends the signal $\psi_i(t) = \psi(t - \frac{iT}{2^k})$.

1. If the pulse is given by the waveform $\psi(t)$ depicted below. What is the value of A that gives us signals of energy equal to one as a function of k and T?



2. We want to transmit the hypothesis $i = 3$ followed by the hypothesis $j = 2^k - 1$. Plot the waveform you will see at the output of the transmitter, using the pulse given in the previous question.

3. Sketch the optimal receiver. What is the minimum number of filters you need for the optimal receiver? Explain.

4. What is the major drawback of this signaling scheme? Explain.