ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 27 | Information Theory and Coding
Homework 12 (Graded DUE 21-12-2010) | December 14, 2010, SG1 – 15:15-17:00

Problem 1. (a) They should observe that by increasing the length, the error probability reduces.

(b) They should see an approximate line with β being the slope of the line. Ideally β should be $\frac{1}{2}$ but they see something less.

Problem 2. It's a straightforward program.

Problem 3. (a) All the information that we have at the output is the bits that are not erased. A little thought shows that the equation $x_I = uG_I$ is just the most general system of equations that we can have using the available information and we can not do better than this.

(b) If $\text{rank}(G_I) < |I|$, then there is more that one solution to the equation $x_I = uG_I$, and all these solutions are equiprobable. So in terms of the MAP decoding we have ambiguity in the decoded vector.

(c) It's a simple program, they can only check the decoding error by looking at the rank of G_I at each time. They should note that ideally p_e is less than what they got in problem 1 (i.e., the upper-bound on the block error probability of the MAP decoder.)

(d) Let $G_n = \left[\begin{smallmatrix} 1 & 0 \\ 1 & 1 \end{smallmatrix}\right]^{\otimes n}$. It can be proven by induction that $G_n^{-1} = G_n$. We have $x = uG_n$ and something important to note here is that the vector u is a N dimensional vector consisting of both the information bits and the fixed bits (which we fix them to zero and are known both to the encoder and decoder). Let I and $\bar{I}$ be the set of non-erased and erased bits respectively. Also let $G_{n,I}$ and $G_{n,\bar{I}}$ be the part of G_n with rows in I and $\bar{I}$ respectively. We have,

$$x = uG_n \Rightarrow u = xG_n \Rightarrow u = x_I G_{n,I} + x_{\bar{I}} G_{n,\bar{I}} \Rightarrow u + x_I G_{n,I} = x_{\bar{I}} G_{n,\bar{I}}.$$

Note that $u + x_I G_{n,I}$ is known to the decoder and we denote it by u'. Assuming $u' = (u'_1, u'_2, \cdots, u'_N)$. To decode the i-th information bit (according to the SC decoder), the decoder has to solve the following set of equations.

$$(u'_1, u'_2, \cdots, u'_{i-1}, u'_i, *, *, \cdots, *) = x_{\bar{I}} G_{n,\bar{I}}, \tag{1}$$

where the value of the bits $u'_1, u'_2, \cdots, u'_{i-1}$ are presumably known to the decoder and the value of the bits $u'_{i+1}, \cdots, u'_N$ are not known to the decoder yet (hence denoted by $*$ in above). So the decoder has to find u_i with this available information. A little thought shows that the decoder can decode u_i iff the i-th column of $G_{n,\bar{I}}$ is linearly dependent with the first $i-1$ columns of $G_{n,\bar{I}}$. Intuitively, if the i-th column is in the span of the first $i-1$ columns of $G_{n,\bar{I}}$, then a linear combination of these $i-1$ columns will result the i-th column and if we perform the same linear combination on the bits $u'_1, \cdots, u'_{i-1}$ we should get u_i. So the decoder has an error iff the i-th column of $G_{n,\bar{I}}$ is not in the span of the first $i-1$ columns of $G_{n,\bar{I}}$. The value of u_i in case of no error can be easily found by writing the set of linear equations in (1) in terms of the row-echelon form of $G_{n,\bar{I}}$. The rest is similar to part (c).