ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 17 Signal Processing for Communications

Homework 10 May 9, 2011, INF 1 - 10:15-12:00

Problem 1. Let $x(t)$ and $y(t)$ $t \in \mathbb{R}$ be complex signals with Fourier transforms $X(f)$ and $Y(f)$. Prove the following properties:

1) $x(t-\tau) \xrightarrow{\text{FT}} e^{-j2\pi\tau f}X(f)$.

2) $X(t) \xrightarrow{\text{FT}} x(-f)$.

3) $x(at) \xrightarrow{\text{FT}} \frac{1}{a}X(\frac{f}{a})$.

4) $(x * y)(t) \xrightarrow{\text{FT}} X(f)Y(f)$.

Problem 2. One of the standard ways of describing the sampling operation relies on the concept of modulation by a pulse train. Choose a sampling interval T_s and define a continuous-time pulse train $p(t)$ as $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$.

The Fourier Transform of the pulse train is $P(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T_s})$.

This is tricky to show, so just take the result as is. The sampled signal is simply the modulation of an arbitrary-continuous time signal $x(t)$ by the pulse train: $x_s(t) = p(t)x(t)$. Note that, now, this sampled signal is still continuous time but, by the properties of the delta function, is non-zero only at multiples of T_s; in a sense, $x_s(t)$ is a discrete-time signal brutally embedded in the continuous time world.

Here is the question: derive the Fourier transform of $x_s(t)$ and show that if $x(t)$ is bandlimited to $\frac{\pi}{T_s}$ then we can reconstruct $x(t)$ from $x_s(t)$.

Problem 3. Assume $x(t)$ is a continuous-time pure sinusoid at 10KHz. It is sampled with a sampler at 8KHz and then interpolated back to a continuous-time signal with an interpolator at 8KHz. What is the perceived frequency of the interpolated sinusoid?

Problem 4. 1) Consider a real continuous-time signal $x(t)$. All you know about the signal is that $x(t) = 0$ for $|t| > t_0$. Can you determine a sampling frequency F_s so that when you sample $x(t)$, there is no aliasing? Explain.

2) We have seen that any discrete-time sequence can be sinc-interpolated into a continuous-time signal which is f_N-bandlimited; f_N depends on the interpolation interval T_s via the relation $f_N = \frac{1}{2T_s}$.

Consider the continuous-time signal $x_c(t) = e^{-\frac{t}{T_s}}$ and the discrete-time sequence $x[n] = e^{-n}$. Clearly, $x_c(nT_s) = x[n]$; but, can we also say that $x_c(t)$ is the signal we obtain if we apply sinc interpolation to the sequence $x[n] = e^{-n}$ with interpolation interval T_s? Explain in detail.

Problem 5. Problem 9.4 in the book.