ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 11 Signal Processing for Communications
Homework 6 March 28, 2011, INF 213 - 10:15-12:00

Problem 1 (Overlap Add and Save Methods)**.** Consider an L-point input sequence $x[n] = rand(1, L)$ and a P-point impulse response

$$h[n] = \begin{cases} \frac{10}{n+10} & 0 \leq n < P-1 \\ 0 & \text{otherwise} \end{cases}$$

i) Use the following code in MATLAB to compute $y[n] = x[n] \circledast h[n]$ for $L = 100000$ and $P = 20$. How long does this computation take?

```
% L is length of input signal and P is length of impulse response.
L=100000, P=20;
x=rand(1,L); % Generate random sequence of numbers between [0,1].
h=10./((0:P-1)+10);
% Direct convolution
tic;
y1=conv(x,h); % Computing convolution of x[n] and h[n].
toc % computing the elapsed time.
```

ii) Segment $x[n]$ to sections of length $B = 50$ as follows:

$$x[n] = \sum_{r=0}^{\infty} x_r[n - rB],$$

where

$$x_r[n] = \begin{cases} x[n + rB] & 0 \leq n \leq B-1 \\ 0 & \text{otherwise} \end{cases}$$

and show theoretically that $y[n] = \sum_{r=0}^{\infty} y_r[n + rB]$ where $y_r[n] = x_r[n] \circledast h[n]$.

iii) Use the following code which is the algorithm mentioned in part (ii) to compute $y[n]$. How long does it take? Compare the resulted elapsed time with the one in part (i).

```
% Overlap-add method :
y2=zeros(SIZE); % Replace the variable SIZE with an appropriate number.
temp=zeros(SIZE);
B=50;
tic;
for i=1:(L/B)
    % each time we consider a B-points window of x, convolve
    %it with h and save the output in B+(P-1) points of temp
    temp( (i-1)*B + 1 : i*B + (P-1) )=conv(x( (i-1)*B + 1 : i*B ),h);
    % add with previous results considering overlaps
    y2=y2+temp;
    % make temp zero
    temp=zeros(SIZE));
end
toc
```

iv) Verify that if a B-point sequence is circularly convolved with a $P < B$-point sequence ($P < L$), then the first $P-1$ points of the result are the only points different from what would be obtained by employing linear convolution.

v) Again divide $x[n]$ into sections of length B so that each input section $x_r[n]$ overlaps the preceding section by $P-1$ points. Call the circular convolution of each segment with $h[n]$, name it $y_{rp}[n]$. Write $y[n] = x[n] \circledast h[n]$ in terms of $y_{rp}[n]$.

Hint: $x_r[n] = x[n + r(B-P+1) - P + 1], 0 \le n \le B-1$.

vi) Use part (v) to compute $y[n]$ for $L = 100000, P = 20, B = 50$. How long does this take?

Problem 2. i) Let $H(e^{i2\pi f})$ be the ideal low pass filter with cut-off frequency f_c, i.e.

$$H(e^{i2\pi f}) = \begin{cases} 1 & |f| \le f_c \\ 0 & otherwise \end{cases}$$

Prove that $h[n] = \frac{sin(2\pi f_c n)}{\pi n}$ for $n \in \mathbb{Z}$.

ii) The following function is a low pass filter with the aforementioned impulse response with cut-off frequency f_c and length N:

```
%Low Pass Filter with cut-off frequency $f_c$ and length $N$:
function H = LowPass(fc,N)

if (N/2 == floor(N/2)) %Check N is even
    h = [sin(2*fc*pi*[-N/2:-1])./([-N/2:-1]*pi)
            ,2*fc,sin(2*fc*pi*[1:N/2-1])./([1:N/2-1]*pi)];
else %N is odd
    h = [sin(2*fc*pi*[-(N-1)/2:-1])./([-(N-1)/2:-1]*pi)
            ,2*fc,sin(2*fc*pi*[1:(N-1)/2])./([1:(N-1)/2]*pi)];
end

H = fft(h,N);
```

The following code plots the magnitude and phase of the frequency response of the above filter for $f_c = 0.2$ and $N = 100$. Explain why it is not an ideal low pass? Does it tend to an ideal filter by increasing N?

Note that fftshift(H) rearranges the outputs of fft to the standard form.

```
H = LowPass(0.2,100);
subplot(2,1,1)
plot((-N/2:N/2-1)/N,abs(fftshift(H)));
subplot(2,1,2)
stem((-N/2:N/2-1)/N,angle(fftshift(H)));
```

iii) Consider the signal $x_0[n] = \sin(0.05n) + \sin(.1n) + \sin(.2n) + \sin(n)$ for $1 \le n \le 100$. Write a code to filter the frequencies larger than 0.1 Hz. Compare your result with $x_{lp}[n] = \sin(0.05n) + \sin(.1n) + \sin(.2n)$. They should be the same.

iv) (Decimation and Interpolation) Consider a system which performs down sampling by factor M , i.e. $x_d[n] = x_{lp}[Mn]$, and then performs up sampling by factor L, i.e.

$$x_u[n] = \begin{cases} x_d[n/L] & \text{if } n = kL \quad \text{for } k \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$

First assume that $M = L = 2$. We want to retrieve $x_{lp}[n]$ from $x_u[n]$. Find an appropriate interpolation function ($h_{int}[n]$) such that $x_{lp}[n] = h_{int}[n] \circledast x_u[n]$.

Hint: Compare the spectrum of $x_u[n]$ and $x_{lp}[n]$.

v) (Aliasing) Find necessary conditions on M and L in order to successfully retrieve $x_{lp}[n]$ from $x_u[n]$.