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Handout 9 Advanced Digital Communications
Homework 6 November 1, 2010

PROBLEM 1. Consider the transmit pulse:

$$g_{TX}(t) = \text{sinc}\left(\frac{t}{T}\right) \cdot \text{sinc}\left(\frac{t}{2T}\right)$$

(a) Prove that it satisfies the Nyquist condition at symbol rate $\frac{1}{T}$

(b) If $g_{TX}(t)$ is used for Nyquist signaling using 8-PSK (the constellation formed by 8 points equally distributed on the unit circle) at 6 Mbit/s, what is the minimum required channel bandwidth?

(c) For the setting in (b), suppose that the complex baseband channel has impulse response $g_C(t) = \delta(t - 0.5T) - \frac{1}{2}\delta(t - 1.5T) + \frac{1}{4}\delta(t - 2.5T)$. What is the minimum number of states in the trellis for MLSE using the Viterbi algorithm?

PROBLEM 2. Consider the noisy ISI channel given by

$$Y_i = X_i + X_{i-1} + Z_i$$

where X_i and Y_i are the channel input and output, respectively at time index i, $\mathbf{Z}$ is a sequence of i.i.d. Gaussian random variables, with zero mean and unit variance and $x_i \in \{-1, 1\}$.

Calculate the symbol-wise MAP estimate of $\mathbf{X}$, using the BCJR algorithm, if the recieved sequence $\mathbf{Y} = [0.28, -0.54, -0.46, 2.26, 1.52]$. You may assume that the channel is in state $+1$ at the beginning and at the end of the sequence. Compare this to the decoding estimate from the MLSE decoder.

PROBLEM 3. Consider the following real channel,

$$\mathbf{y} = \mathbf{h}x + \mathbf{z},$$

where $x \in \mathbb{R}$ is a random variable, with $E[x] = 0$ and $E[x^2] = \mathcal{E}$, $\mathbf{h}$ is a fixed real vector, and $\mathbf{z}$ is a zero-mean random vector with covariance matrix I, chosen independently of the value of x.

(i) An estimator $\hat{x} = \mathcal{F}(\mathbf{y})$ is said to be *unbiased* if $E[\hat{x}|x] = x$.

(1) Consider the mentioned channel, what is the constraint for a linear estimator, i.e., $\hat{x} = \mathbf{a}^t\mathbf{y}$ to be unbiased?

(2) Find the unbiased linear estimator that minimizes the mean squared error:

$$\sigma^2_{\text{unbiased}} = E[(x - \hat{x})^2]$$

and the value of $\sigma^2_{\text{unbiased}}$ for this estimator.
Hint. By the Cauchy-Schwartz inequality, $(\mathbf{a}^t\mathbf{a})(\mathbf{h}^t\mathbf{h}) \geq |\mathbf{a}^t\mathbf{h}|^2$.

(ii) In this part, we don't restrict ourselves to unbiased estimators. Suppose $\mathcal{F}$ is a linear estimator. Find the linear estimator that minimizes the mean squared error $\sigma^2 = E[(x - \hat{x})^2]$ and the value of σ^2 for this estimator.

Hint. Consider $\hat{x} = \mathbf{a}^t\mathbf{y}$, and minimize σ^2 with respect to vector $\mathbf{a}$. Do the minimization in two steps, first assume $\mathbf{a}^t\mathbf{h} = c$, and minimize σ^2 with respect to vector $\mathbf{a}$ with the constraint $\mathbf{a}^t\mathbf{h} = c$, and then minimize the result with respect to c.

(iii) Compare the two "signal to noise ratio"s $\mathcal{E}/\sigma^2_{\text{unbiased}}$ and $\mathcal{E}/\sigma^2$.

(iv) Now assume x is equally likely to be $+1$ or -1. Suppose a decision is made by quantizing the estimate $\hat{x}$ from either part (i) or (ii) to ± 1. Which estimator would you choose to minimize the probability of error?