ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

School of Computer and Communication Sciences

Handout 21 Signal Processing for Communications

Homework 9 April 26, 2010

PROBLEM 1. One of the standard ways of describing the sampling operation relies on the concept of modulation by a pulse train. Choose a sampling interval T_s and define a continuous-time pulse train $p(t)$ as

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

The Fourier Transform of the pulse train is

$$P(j\Omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\Omega - \frac{k}{T_s})$$

This is tricky to show, so just take the result as is. The sampled signal is simply the modulation of an arbitrary-continuous time signal $x(t)$ by the pulse train:

$$x_s(t) = p(t)x(t)$$

Note that, now, this sampled signal is still continuous time but, by the properties of the delta function, is non-zero only at multiples of T_s; in a sense, $x_s(t)$ is a discrete-time signal brutally embedded in the continuous time world. Here is the question: derive the Fourier transform of $x_s(t)$ and show that if $x(t)$ is bandlimited to $\frac{\pi}{T_s}$ then we can reconstruct $x(t)$ from $x_s(t)$.

PROBLEM 2. Assume $x(t)$ is a continuous-time pure sinusoid at $10KHz$. It is sampled with a sampler at $8KHz$ and then interpolated back to a continuous-time signal with an interpolator at $8KHz$. What is the perceived frequency of the interpolated sinusoid?

PROBLEM 3. For your birthday, you receive an unexpected present: a $4MHz$ A/D converter, complete with anti-aliasing filter. This means you can safely sample signals up to a frequency of $2MHz$; since this frequency is above the AM radio frequency band, you decide to hook up the A/D to your favorite signal processing system and build an entirely digital radio receiver. In this exercise we will explore how to do so.

Simply, assume that the AM radio spectrum extends from $1Mhz$ to $1.2Mhz$ and that in this band you have ten channels side by side, each one of which occupies $20KHz$.

(1) Sketch the digital spectrum at the output of the A/D converter, and show the bands occupied by the channels, numbered from 1 to 10, with their beginning and end frequencies.

The first thing that you need to do is to find a way to isolate the channel you want to listen to and to eliminate the rest. For this, you need a bandpass filter centered on the band of interest. Of course, this filter must be tunable in the sense that you must be able to change its spectral location when you want to change station. An easy way to obtain a tunable bandpass filter is by modulating a lowpass filter with a sinusoidal oscillator whose frequency is controllable by the user:

(1) As an example of a tunable filter, assume $h[n]$ is an ideal lowpass filter with cutoff frequency $\frac{\pi}{8}$. Plot the magnitude response of the filter $h_m[n] = \cos(\omega_m n)h[n]$, where $\omega_m = \frac{\pi}{2}$; ω_m is called the tuning frequency.

(2) Specify the cutoff frequency of a lowpass filter which can be used to select one of the AM channels above.

(3) Specify the tuning frequencies for channel 1, 5 and 10.

Now that you know how to select a channel, all that is left to do is to demodulate the signal and feed it to a D/A converter and to a loudspeaker.

(1) Sketch the complete block diagram of the radio receiver, from the antenna going into the A/D converter to the final loudspeaker. Use only one sinusoidal oscillator. Do not forget the filter before the D/A (specify its bandwidth).

The whole receiver now works at a rate of $4MHz$; since it outputs audio signals, this is clearly a waste.

(1) Which is the minimum D/A frequency you can use? Modify the receivers block diagram with the necessary elements to use a low frequency D/A.

PROBLEM 4. Consider a real continuous-time signal $x(t)$. All you know about the signal is that $x(t) = 0$ for $|t| > t_0$. Can you determine a sampling frequency F_s so that when you sample $x(t)$, there is no aliasing? Explain.

PROBLEM 5. We have seen that any discrete-time sequence can be sinc-interpolated into a continuous-time signal which is Ω_N-bandlimited; Ω_N depends on the interpolation interval T_s via the relation $\Omega_N = \frac{\pi}{T_s}$.

Consider the continuous-time signal $x_c(t) = e^{-\frac{t}{T_s}}$ and the discrete-time sequence $x[n] = e^{-n}$. Clearly, $x_c(nT_s) = x[n]$; but, can we also say that $x_c(t)$ is the signal we obtain if we apply sinc interpolation to the sequence $x[n] = e^{-n}$ with interpolation interval T_s? Explain in detail.