ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
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Handout 11 Signal Processing for Communications
Homework 7 April 12, 2010

Problem 1 (Excercise 6.4 of textbook). (I) Consider a causal LTI system with the following transfer function:

$$H(z) = \frac{3 + 4.5z^{-1}}{1 + 1.5z^{-1}} - \frac{2}{1 - 0.5z^{-1}}$$

Sketch the pole-zero plot of the transfer function and specify its region of convergence. Is the system stable?

(II) Consider the transfer function of an anticausal LTI system

$$H(z) = (1 - z^{-1}) - \frac{1}{1 - 0.5z^{-1}}$$

Sketch the pole-zero plot of the transfer function and specify its region of convergence. Is the system stable?

Problem 2 (Excercise 7.2 of textbook). Assume $\mathcal{G}$ is stable, causal IIR filter with impulse response $g[n]$ and transfer function $G(z)$. Which of the following statements is/are true for any choice of $G(z)$?

(a) The inverse filter, $1/G(z)$, is stable.

(b) The inverse filter is FIR.

(c) The DTFT of $g[n]$ exists.

(d) The cascade $G(z)G(z)$ is stable.

Problem 3 (Excercise 7.3 of textbook). Consider $G(z)$, the transfer function of a causal stable LTI system. Which of the following statements is/are true for any such $G(z)$?

(a) The zeros of $G(z)$ are inside the unit circle.

(b) The ROC of $G(z)$ includes the curve $|z| = .5$.

(c) The system $H(z) = (1 - 3z^-1)G(z)$ is stable.

(d) The system is an IIR filter.

Problem 4 (Excercise 7.9 of textbook). Consider a causal IIR filter with the following transfer function:

$$H(z) = \frac{1 + z^-1}{1 - 1.6\cos(2\pi/7)z^-1 + .64z^{-2}}$$

(a) Sketch the pole-zero plot of the filter and the ROC of its transfer function.

(b) Sketch the magnitude of its frequency response.

(c) Draw at least two different block diagrams which implement the filter.

(d) Compute the first five values of the signal $y[n] = h[n] * x[n]$, where $x[n] = \delta[n] + 2\delta[n-1]$. Assume zero initial conditions.

PROBLEM 5 (EXCERCISE 7.11 OF TEXTBOOK). In data communication systems over phone lines (such as voiceband modems), one of the major problems is represented by echos. Impedance mismatches along the analog line create delayed and attenuated replicas of the transmitted signal. These replicas are added to the original signal and represent one type of distortion.

Assume a simple situation where a single echo is created; the transmitted signal is $x[n]$ and, because of the echo, the received signal is

$$y[n] = x[n] - \alpha x[n-D]$$

where α is the attenuation factor (with $0 < \alpha < 1$) and D is the echo delay (assume D is an integer).

(a) Write the transfer function $H(z)$ of the echo system, i.e. the system which produces $y[n]$ from $x[n]$.

(b) Sketch the pole-zero plot for $H(z)$ for $\alpha = 0.1$ and $D = 12$ (for our purposes, assume $(0.826)^{12} = 0.1$).

(c) Sketch the squared magnitude response $|H(e^{j\omega})|^2$.

Now assume we have a good estimate of α and D; we want to design a causal echo cancellation filter, i.e. a filter with causal impulse response $g[n]$ such that $y[n] * g[n] = x[n]$.

(d) Write the expression for $G(z)$.

(e) Sketch its pole-zero plot and its ROC for the same values of α and D as before.

(f) What is the practical difficulty in implementing this echo cancellation system?