ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
School of Computer and Communication Sciences

Handout 7 Signal Processing for Communications
Homework 5 March 22, 2010

PROBLEM 1. By direct computation of convolution sum, determine the step response of a linear time invariant system whose impulse response is

$$h[n] = a^{-n}u[-n], \qquad 0 < a < 1.$$

PROBLEM 2. If $x[n]$ is zero except for N consecutive points, and $h[n]$ is zero, except for M consecutive points, what is the maximum number of consecutive points for which $y[n]$, the output to the LTI system with impulse response $h[n]$, can be nonzero?

PROBLEM 3. Consider a discrete-time LTI system with frequency response $H(e^{j\omega})$ and corresponding impulse response $h[n]$.

(a) We are given the following three clues about the system:

(i) The system is causal.

(ii) $H(e^{j\omega}) = H^*(e^{-j\omega})$.

(iii) The DTFT of the sequence $h[n+1]$ is real.

Use these three clues to show that the system has an impulse response of finite duration.

(b) In addition to the preceding three clues, we are now given two more clues:

(iv) $\frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, d\omega = 2$.

(v) $H(e^{j\pi}) = 0$.

Is there enough information to identify the system uniquely? If so, determine the impulse response $h[n]$. If not, specify as much as you can about $h[n]$.

PROBLEM 4. Let $x[n]$ and $y[n]$ denote complexe sequences and $X(e^{j\omega})$ and $Y(e^{j\omega})$ their respective Fourier transforms.

(a) Determine, in terms of $x[n]$ and $y[n]$, the sequence whose Fourier transform is $X(e^{j\omega})Y^*(e^{j\omega})$.

(b) Show that

$$\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})\, d\omega$$

(c) Determine the numerical value of the sum

$$\sum_{n=-\infty}^{\infty} \frac{\sin(\pi n/4)}{2\pi n}\frac{\sin(\pi n/6)}{5\pi n}$$

PROBLEM 5. For the following sequences $x[n]$, determine the z transform $X(z)$ and its region of convergence (ROC). Sketch the pole/zero diagram in the complex z plane.

(i) $x[n] = a^n u[n] + b^n u[n] + c^n u[-n-1], \quad |a| < |b| < |c|$.

(ii) $x[n] = \frac{(-1)^n}{n!} u[n]$.

(iii) $x[n] = -\frac{1}{n} u[n-1]$.

PROBLEM 6. Use the z transform to analyze the causal LTI system implemented by the following recursive difference equation: $y[n] = x[n] - \frac{1}{15} y[n-1] + \frac{2}{5} y[n-2]$.

(i) Determine the transfer function $H(z) = \frac{Y(z)}{X(z)}$.

(ii) Sketch the poles and zeros in the complex z plane and give the region of convergence of $H(z)$. Is the system stable?

(iii) Does the Fourier transform $H(e^{j\omega})$ converge?

(iv) Determine the impulse response $h[n]$ of the causal LTI system.