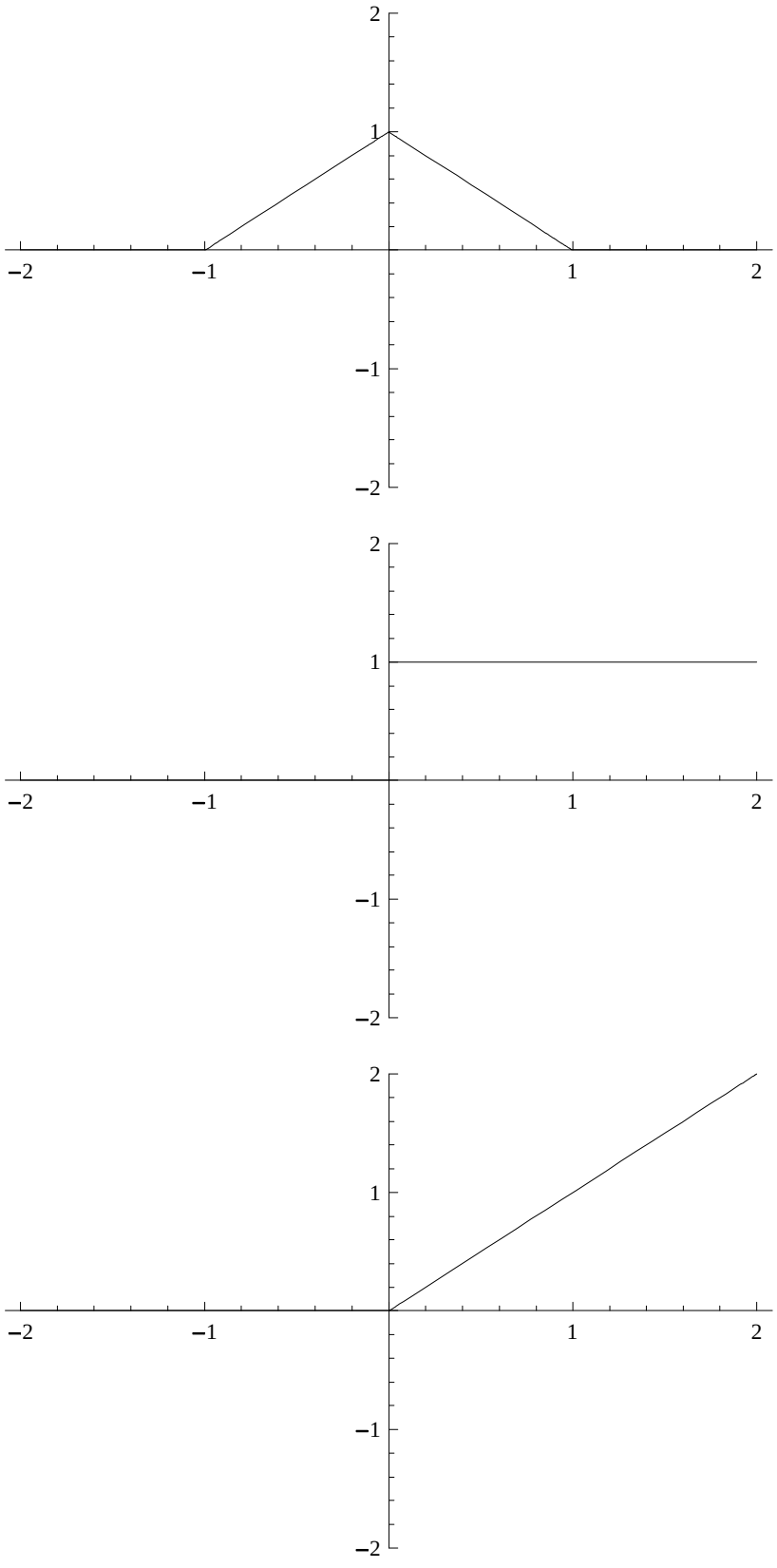
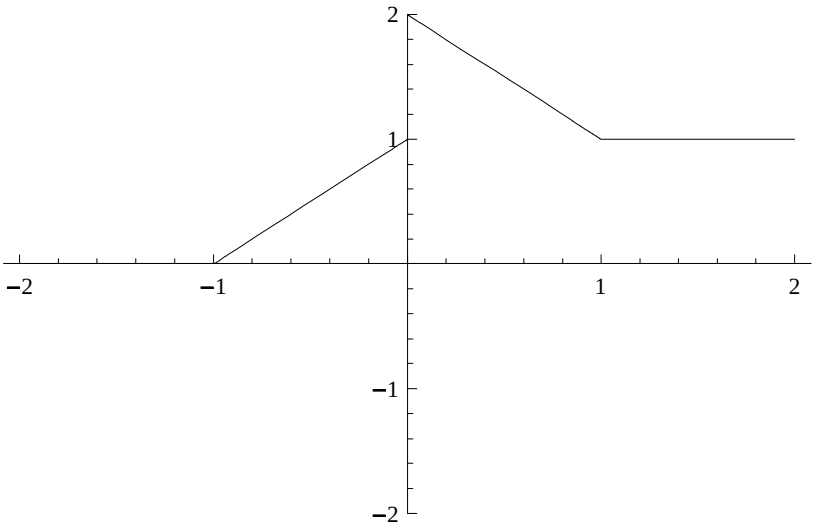
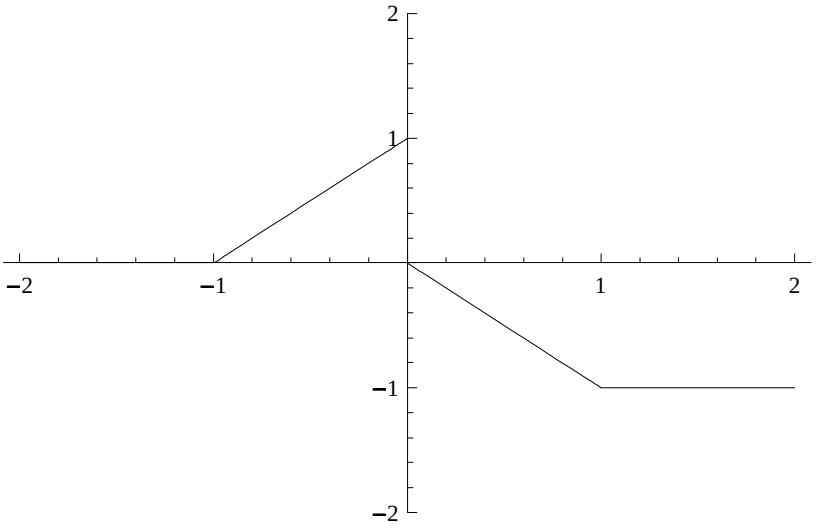
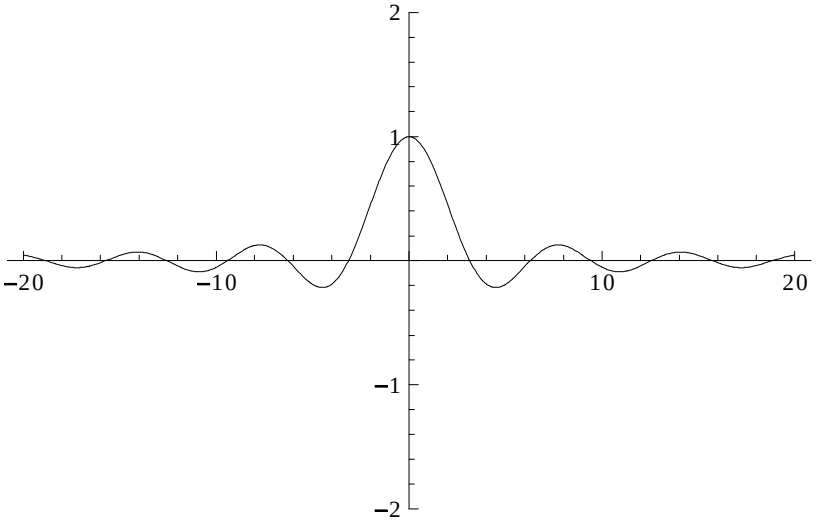
Problem 1




2
1
−2
−1
1
2
−1
−2


2
1
−2
−1
1
2
−1
−2


2
1
−20
−10
10
20
−1
−2

Problem 2

$$A\sin\left(2\pi vt+\phi\right)=5\cos\left(10t+\frac{\pi}{2}\right)=5\sin\left(\frac{5}{\pi}\left(2\pi t\right)+\pi\right)$$

$A=5$, $T=\pi/5$, $\phi=\pi$.

Problem 3

a) We have

$$0=A_1\left(\sin\left(2\pi v_1t+\phi_1\right)-\sin\left(2\pi v_1\left(t+T\right)+\phi_1\right)\right)+A_2\left(\sin\left(2\pi v_2t+\phi_2\right)-\sin\left(2\pi v_2\left(t+T\right)+\phi_2\right)\right)$$

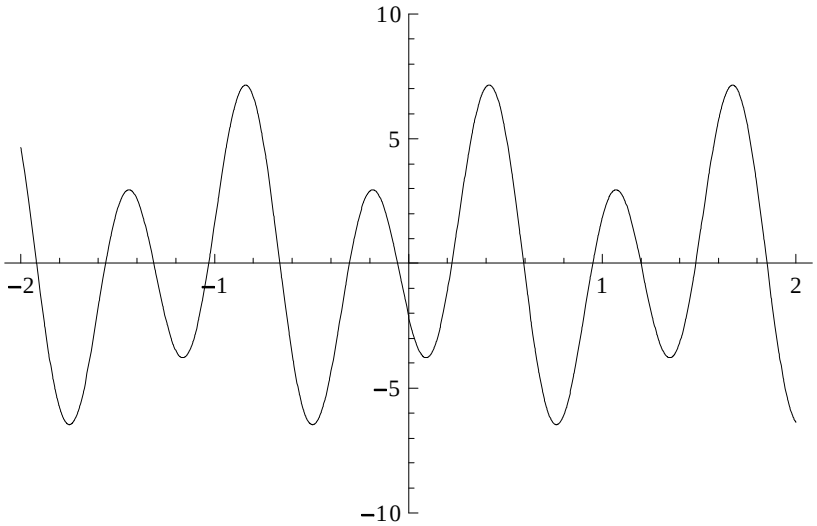
Then

$$T=\frac{p}{v_1}\text{ and }T=\frac{q}{v_2}$$

where $p,q\in\mathbb{N}$ and thus leads to the following condition

$$\frac{v_1}{v_2}=\frac{p}{q}\in\mathbb{Q}$$

b) $v_1=\frac{10}{2\pi}$, $v_2=\frac{5}{2\pi}$, $\frac{v_1}{v_2}=\frac{p}{q}=\frac{2}{1}$ then $T=\frac{p}{v_1}=\frac{2\pi}{5}$.



Problem 4

a) By change of integration variable $\tau=t-s$ we get

$$\begin{aligned}\left(x*y\right)\left(t\right) &= \int_{-\infty}^{\infty}x\left(s\right)\ y\left(t-s\right)ds\\ &= \int_{\infty}^{-\infty}x\left(t-\tau\right)\ y\left(\tau\right)\left(-d\tau\right)\\ &= \int_{-\infty}^{\infty}x\left(t-\tau\right)\ y\left(\tau\right)d\tau\\ &= \left(y*x\right)\left(t\right)\end{aligned}$$

b)

$$\begin{aligned}
\sum_{t=-\infty}^{\infty} z_t &= \sum_{t=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} x_s \; y_{t-s} \\
&= \sum_{s=-\infty}^{\infty} \sum_{t=-\infty}^{\infty} x_s \; y_{t-s} \\
&= \sum_{s=-\infty}^{\infty} x_s \sum_{t=-\infty}^{\infty} y_{t-s} \\
&= \sum_{s=-\infty}^{\infty} x_s \sum_{\tau=-\infty}^{\infty} y_{\tau}
\end{aligned}$$

Thus

$$\begin{aligned}
0 &= \sum_{t=-\infty}^{\infty} z_t - \sum_{s=-\infty}^{\infty} x_s \\
&= \sum_{s=-\infty}^{\infty} x_s \sum_{\tau=-\infty}^{\infty} y_{\tau} - \sum_{s=-\infty}^{\infty} x_s \\
&= \sum_{s=-\infty}^{\infty} x_s \left(\sum_{\tau=-\infty}^{\infty} y_{\tau} - 1 \right)
\end{aligned}$$

and

$$\sum_{\tau=-\infty}^{\infty} y_{\tau} = 1$$