Handout 7 Introduction to Communication Systems
Homework 4 October 8, 2009

Problem 1

Sketch the following signals

$$\text{Triangle}\,(t) = \begin{cases} 0 & \text{if} \quad |t| > 1 \\ 1 - |t| & \text{if} \quad |t| \leq 1 \end{cases}$$

$$\delta_{-1}\,(t) = \begin{cases} 0 & \text{if} \quad t < 0 \\ 1 & \text{if} \quad t \geq 0 \end{cases}$$

$$\delta_{-2}\,(t) = \int_{-\infty}^{t} \delta_{-1}\,(\tau)\;d\tau$$

$$\text{Sum}\,(t) = \text{Triangle}\,(t) + \delta_{-1}\,(t)$$

$$\text{Diff}\,(t) = \text{Triangle}\,(t) - \delta_{-1}\,(t)$$

$$\text{Sinc}\,(t) = \begin{cases} 1 & \text{if} \quad t = 0 \\ \frac{\sin \pi t}{\pi t} & \text{if} \quad t \neq 0 \end{cases}$$

Problem 2

Specify the amplitude, frequency and phase of the signal:

$$x\,(t) = 5\cos\left(10t + \frac{\pi}{2}\right)$$

What is the period of $x(t)$?

Problem 3

a) We know that a continuous-time sinusoid is a periodic signal. Is the sum of two sinusoids also periodic? Under which conditions? What is the period?

b) Sketch $x\,(t) = 5\cos\,(10t + 2) + 2.5\sin\,(5t)$. Show that $x\,(t)$ is periodic. Which is the period?

a) Let $x, y : \mathbb{R} \to \mathbb{R}$ be two real functions. We define a new function $z(t) = (x * y)(t)$ where

$$(x * y)(t) := \int_{-\infty}^{\infty} x(s) \; y(t-s)\, ds$$

this operation is called the convolution product of x and y.

Show that $(x * y)(t) = (y * x)(t)$.

b) For discrete time namely for $x, y : \mathbb{Z} \to \mathbb{R}$, we define the convolution by

$$(x * y)_t = \sum_{s=-\infty}^{\infty} x_s \; y_{t-s}.$$

Defining $z_t = (x * y)_t$, for which condition on y we have $\sum_{t=-\infty}^{\infty} z_t = \sum_{s=-\infty}^{\infty} x_s \neq 0$?