

This is your first graded homework. Your grade will solely be evaluated based on your answers, not your friends' answers, nor your look or gender ;). Try to write down your answers as clear as possible such that at least one of the assistants would be able to read them. :)

PROBLEM 1. Assume that we use the following code for encoding the source stream. Let  $x = x_1x_2x_3, \dots$  be a long sequence of source symbols where  $x_i \in \{a, b, c, d\}$ . Let  $\mathbf{c} = c(x_1)c(x_2)c(x_3)\dots$  be the corresponding long binary sequence which results when we encode  $x$ . Imagine picking one bit at random from  $c$ . What is the probability that this bit is a 1?

Symbol	Code	probability
a	0	1/2
b	10	1/4
c	110	1/8
d	111	1/8

PROBLEM 2. Find a probability distribution  $\{p_1, p_2, p_3, p_4\}$  such that there are *two* optimal codes that assign different lengths  $l_i$  to the four symbols  $a, b, c, d$ .

Symbol	probability	length
$a$	$p_1$	$l_1$
$b$	$p_2$	$l_2$
$c$	$p_3$	$l_3$
$d$	$p_4$	$l_4$

PROBLEM 3. The World Series is a seven-game series that terminates as soon as either Nadal or Federer wins four games. Let  $X$  be the random variable that represents the outcome of a World Series between Nadal (N) and Federer (F). Possible values of  $X$  are FFFF, NFNFNFN, NNNFFFF, ... . Let  $Y$  be the number of games played which ranges from 4 to 7. Assume that Nadal and Federer are equally matched and that the games are independent. Calculate  $H(X)$  and  $H(Y)$

PROBLEM 4. Which of these codes cannot be Huffman codes for any probability assignment?

- $\{0, 10, 11\}$ .
- $\{00, 01, 10, 110\}$ .
- $\{01, 10\}$ .

PROBLEM 5. Consider the following source.

Symbol	probability
a	1/2
b	1/4
c	1/8
d	1/8

- What is the entropy of this source?
- Find a binary Huffman code. What is the average code length?
- Find a Shannon-Fano code. What is the average code length?